

## How High Does a Ball Bounce? – ID: XXXX

By Lauren Jensen

Time required  
45 minutes

## Activity Overview

*Students are introduced to modeling exponential data through an investigation of the rebound heights of a racquetball bouncing. Students use multiple representations to explore aspects of the exponential equation that models the data.*

## Concepts

- *Data representation and interpretation*
- *Exponential equations and regressions*

## Teacher Preparation

*This investigation offers opportunities for review and consolidation of key concepts related to exponential functions. As such, care should be taken to provide ample time for ALL students to engage actively with the requirements of the task, allowing some who may have missed aspects of earlier work the opportunity to build a new and deeper understanding.*

- *At the Algebra 1 level, this activity can serve to consolidate earlier work on exponential functions. It offers a suitable introduction to exploring exponential data, model fitting using exponential functions, and interpretation of graphs.*
- *Begin by reviewing with students the general exponential form of  $y = ab^x$ .*
- *The screenshots on pages 2 and 3 demonstrate expected student results. Refer to the screenshots on page 4 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “XXXX” in the quick search box.***

## Classroom Management

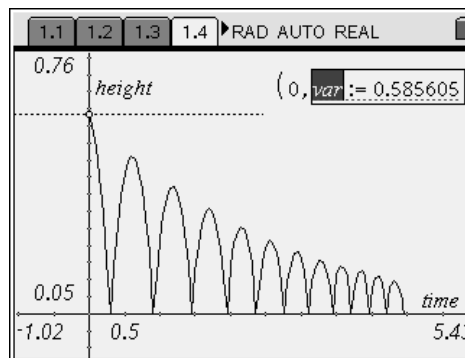
- *This activity is intended to be mainly **teacher-led**, with students in small groups. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *The student worksheet Alg1Act05\_BallBounce\_worksheet\_EN helps guide students through the activity via a series of questions. It also provides a place for students to record their answers.*
- *The TI-Nspire solution document (.tns file) Alg1Act05\_BallBounce\_Soln\_EN shows the expected results of working through the activity.*
- *Suggestions for optional extension questions are provided at the end of this activity.*

## TI-Nspire™ Applications

*Graphs & Geometry, Lists & Spreadsheet, Notes*

This investigation has students explore data that was collected by dropping a racquetball from waist height and letting it bounce repeatedly. A motion detector recorded the ball's height (in meters) versus time (in seconds). After completing the activity, students will be able to develop exponential models and make conjectures based on interpolation and extrapolation of the data.

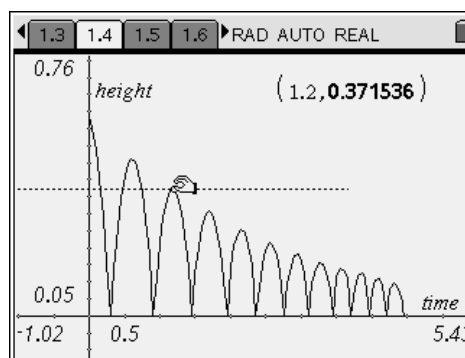
**Step 1:** On page 1.4, students will find a graph of height of the ball with respect to time. An open point on the graph has been placed on the graph, and its coordinates are displayed in the upper right corner. Students should identify the  $y$ -coordinate as the height of the ball, and they will store its value as the variable **ht**.



**Step 2:** When students move to page 1.4, they will find the numbers 0 through 11 in Column A, representing the number of each bounce. In the formula cell for Column B, students will collect the heights for each bounce number. Have them select **MENU > Data > Data Capture > Manual Data Capture** and enter **ht** as the variable to be captured.

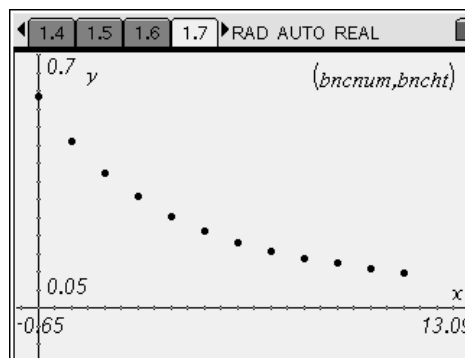
1.3 1.4 1.5 1.6 RAD AUTO REAL			
A	bncnum	B	bncht
			$\text{capture}(\text{ht}, \uparrow)$
1	0		
2	1		
3	2		
4	3		
5	4		
$\text{bncht} := \text{capture}(\text{ht}, 0)$			

**Step 3:** Returning to page 1.4, make sure that the point on the graph is set at the initial value (when  $\text{time} = 0$ ). Students will press  $\text{ctrl} + \text{[point icon]}$  to capture the initial height into Column B. Next, they should grab the point, move it to the next "peak" in the graph and capture the height of the first bounce. Have them continue until all of the rebound heights have been captured in Column B.



**Step 4:** On page 1.5, students will create the scatter plot (*bncnum*, *bncht*) by selecting **MENU > Graph Type > Scatter Plot**. After displaying the scatter plot, pressing  $\text{ctrl} + \text{G}$  will hide the Entry Line, freeing up space on the screen.

Have a discussion about why an exponential function with best model this data.

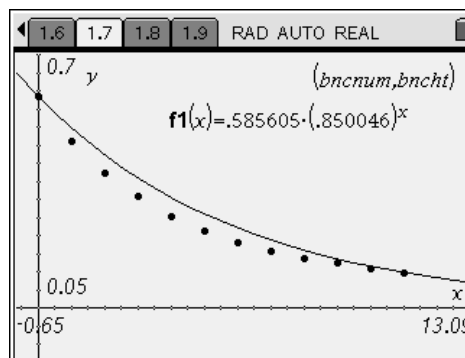


**Step 5:** Next, students should use the collected heights to find the average ratio or rebound rate percentage of the racquet ball, in an effort to ultimately derive an exponential equation that models the data. In the spreadsheet on page 1.6, students should enter the formula **=b2/b1** into cell C1 (*not* in the formula cell for Column C). With cell C1 active, select **MENU > Data > Fill Down**, arrow down to cell C10, and press  $\text{enter}$ . This populates the column with the ratios between successive rebound heights. To find the average ratio, use the **mean**( command in cell D1.

num	B	bncht	C	ratios	D
			=capture(ht,0)		
1	0	.585605	.790396		.850046
2	1	.46286	.802696		
3	2	.371536	.828081		
4	3	.307662	.827031		
5	4	.254446	.839153		
			D1   =mean(ratios)		

**Step 6:** Now that the students have found the average ratio of the rebound height, have them recall the initial height of the ball. Guide them to use these two values to create an exponential equation to model the data. On page 1.7, select **MENU > Graph Type > Function** and define **f1** to be of the form:

$$f1(x) = \text{InitialHeight} \cdot (\text{AverageRatio})^x$$



Students now have the information they need to explore the exponential model of the data. Have them complete the exercises on the student worksheet.

### Extensions

- Students may perform an exponential regression of the data and compare it to the one obtained by finding the average ratio.
- Have students compare the rebound rates of a different type of ball, such as a volleyball, baseball, etc.

### Student Worksheet Solutions

- Answers will vary. Accept reasonable explanations.
- 0.585605 m
- 0.850046
- Answers will vary. Accept reasonable explanations.
- 14.9954%
- $f1(x) = 0.585605 \cdot (0.850046)^x$
- 5th bounce: 0.259906 m; 16th bounce: 0.043519 m
- 10th bounce
- Answers will vary. Accept reasonable explanations.

### How High Does a Ball Bounce?– ID: XXXX

(Student)TI-Nspire File: *Alg1Act05\_BallBounce\_EN.tns*

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

**HOW HIGH DOES A BALL BOUNCE?**

**Algebra 1**

Data and exponential modeling

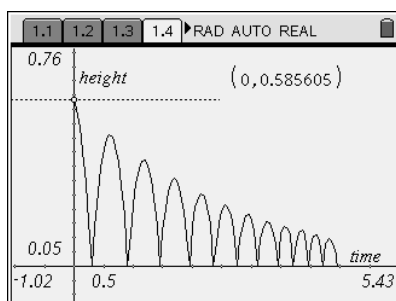
1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

This investigation explores data that was collected by dropping a racquetball from waist height and letting it bounce repeatedly. A motion detector recorded the ball's height (in meters) versus time (in seconds). You will find the height of each bounce, make a scatter plot of the data, and create an exponential equation to model the data.

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

The graph on the next page shows the height of the racquetball with respect to time. The open point can be dragged along the graph and its coordinates can be used to identify the ball's height at any given time.

Store the point's  $y$ -coordinate, the height of the ball, as the variable **ht**.



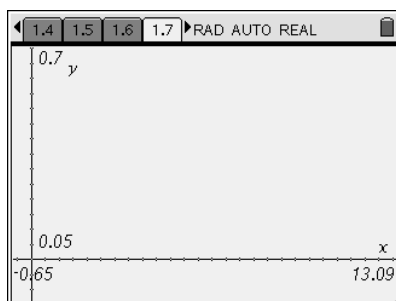
1.2 1.3 1.4 1.5 ▶RAD AUTO REAL

Next, you will collect the rebound height of each bounce in the spreadsheet on the next page. In the formula cell for the **bncht** column, set up a manual data capture for the variable **ht**. Then return to page 1.4 and press **CTRL+ENTER** to capture each maximum height.

Make a scatter plot of the data on page 1.7.

1.3 1.4 1.5 1.6 ▶RAD AUTO REAL

	A bncnum	B bncht	C ratios	D
1	0			
2	1			
3	2			
4	3			
5	4			
6				
A7				



1.5 1.6 1.7 1.8 ▶RAD AUTO REAL

Return to page 1.6.

To populate the **ratios** column with the ratios between success rebound heights, enter **=b2/b1** into cell C1. With that cell selected, choose **Fill Down** from the Data menu, arrow down to cell C10, and press **ENTER**.

In cell D1, calculate the average of these ratios using the formula **=mean(ratios)**.

1.6 1.7 1.8 1.9 ▶RAD AUTO REAL

Return to the graph on page 1.7 and select **Function** from the Graph Type menu. Use the average ratio you just found and the initial height of the ball to write an exponential function for **f1**. Then answer the questions on your worksheet.