

How Random! – ID: 8555

By Lynne Plettenberg

Time required
90 minutes

Activity Overview

In this activity, students use simulations and graphs to explore the common sense notion that repeatedly flipping a coin results in “heads up” about half of the time. First, they simulate an experiment by representing single coin flips with random numbers. Next, they use a given formula to simulate multiple coin flips at once, allowing them to perform a large number of trials easily. Lastly, they create a histogram of their data, observing that the data clusters around the mean.

Concepts

- *Simulations*
- *Binomial Experiments*
- *Law of Large Numbers*

Teacher Preparation

This activity is designed for use in an Algebra 2 or Statistics classroom. It is best used to introduce the concepts of data clustering about the mean, setting up a framework for future explorations of normal distributions.

- *Prior to beginning the activity, students should know how to interpret scatter plots and histograms and be familiar with binomial probability and the concept of mean as expected value.*
- *The screenshots on pages 2–3 (top) demonstrate expected student results. Refer to the screenshots on pages 3 (bottom) and 4 for a preview of the student .tns file.*
- ***To download the .tns file, go to <http://education.ti.com/exchange> and enter “8555” in the search box.***

Classroom Management



- *This activity is intended to have students explore **individually and in pairs** (with the teacher providing assistance when needed). However, an alternate approach would be to use the activity in a whole-class format. By using the computer software and the questions found in the student .tns file, you can lead an interactive class discussion on simulating coin flips.*
- *For new users of the handheld, it would be beneficial to demonstrate how to create scatter plots and histograms from data lists.*
- *The .tns file titled StatAct04_HowRandom_Soln_EN shows the expected results of working through the activity.*

TI-Nspire™ Applications

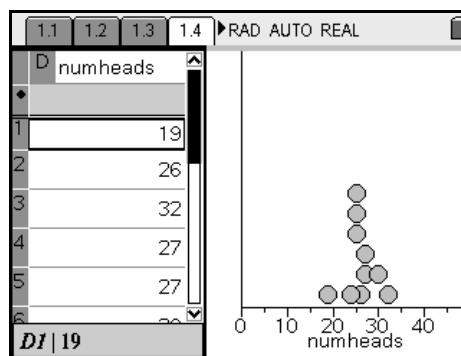
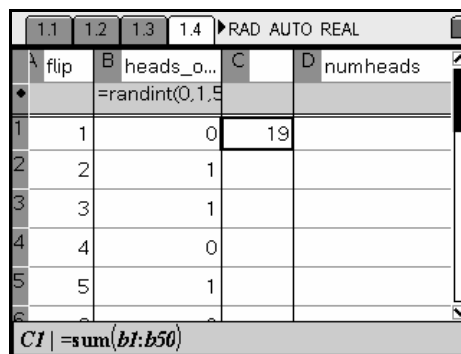
Graphs & Geometry, Lists & Spreadsheet, Notes, Data & Statistics

On page 2.1, students first use the **Fill Down** command from the Data menu to place the numbers 1-50 in Column A, representing the flips. For Column B, guide students as needed in writing the formula **=RandInt(0,1,50)** in the gray formula cell to generate a list of 50 random 0s (tails) and 1s (heads). Discuss how this models the situation.

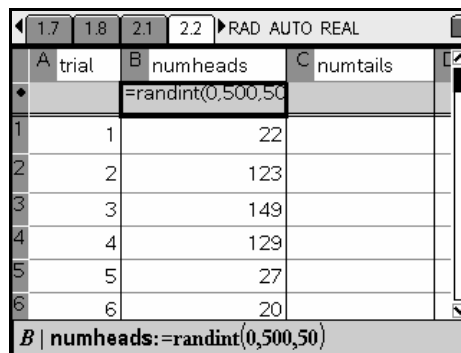
Next, students are prompted to write a formula (**=sum(b1:b50)**) for finding the total number of heads for the 50 flips in cell C1, as shown.

Recording the calculated value of the sum in cell D1, students should then “repeat the experiment” by selecting Column B and choosing **Recalculate** from the Actions menu (or by pressing  + ). The sum in cell C1 will update as the random numbers change; students should record at least 10 such sums in Column D.

When they have done so, tell them to select Column D and choose **MENU > Data > Quick Graph**, which displays a dot plot of the data. Setting the Xmin at 0 and the Xmax at 50, students will find that the data clusters “around the middle.”



In this problem, students are presented with a more complicated situation. This time, they will simulate flipping 500 coins, 50 times! It may appear that the formula to use to generate values for each trial is as simple as randomly choosing 50 numbers from 0 to 500. Students are prompted to use this formula, **=RandInt(0,500,50)**, to generate their trials. It should be immediately obvious to students that this formula is not correct—based on the results Problem 1, most of the data should be clustered around 250. This is not the case, as the data is evenly distributed across the interval 0 to 500.



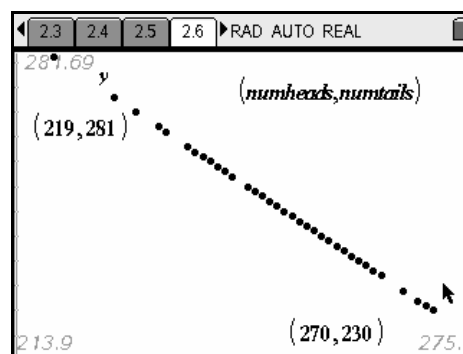
The function $f(x)$ has been defined to correctly represent the situation. Students should enter **=f(A1)** in cell B1 and **Fill Down** to populate the other trials. (**Note:** The formula must be entered in the cells directly; **not** in the gray formula cell.)

Students should easily identify the function **=50-B[]** as the number of tails corresponding to each trial. This may be entered in the formula cell for Column C to populate the column.

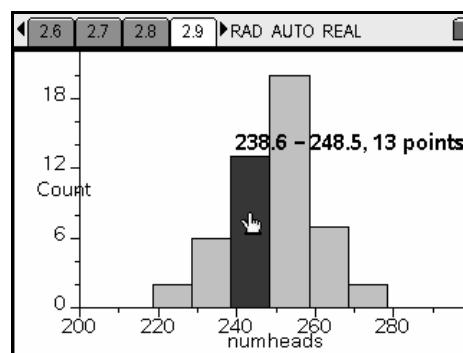
A	B	C
trial	numheads	numtails
1	266	234
2	266	234
3	243	257
4	260	240
5	227	273
6	259	241

Next, students will create the scatter plot (*numheads*, *numtails*). While each student's scatter plot is unique, all should find that the data is clustered around "the middle" (i.e., 250).

Selecting **MENU > Window > Zoom – Data** provides a closer view of the data, where students are asked to describe the 'new' distribution, as well as the maximum and minimum values.



Lastly, students make a histogram of the **numheads** data. The majority of the data should fall around 250, as expected. The mean of the data set is roughly 250, and the standard deviation is about 10. Students are prompted to observe this, as they are asked for the *percent* of values that fall within 240–260, and 230–270—or 1 and 2 standard deviations from the mean! This helps to build a foundation for the understanding the empirical rule of normal distributions.



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(Student)TI-Nspire File: *StatAct04_HowRandom_EN.tns*

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

HOW RANDOM!

Statistics

Simulating coin flips

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

Suppose you flipped a coin 50 times. About how many times would you expect the coin to land heads up?

Most people answer "about 25." The number of heads might be slightly more or less, but it will always be *close* to 25.

1.1 1.2 1.3 1.4 ▶RAD AUTO REAL

But why? Why is it unlikely that you flip only 2 heads or even 48 heads in 50 flips? If flipping a coin is random, shouldn't any number of heads be equally likely?

On the next page, you will simulate flipping a coin. Select cells A1 and A2. Then **Fill Down** to simulate flipping the coin 50 times.

1.1	1.2	1.3	1.4	RAD AUTO REAL		
A	flip	B	heads_o...	C	D	numheads
1	1					
2	2					
3						
4						
5						
6						
Af						

Next, we need to simulate "getting heads or tails." In the formula cell for Column B, use the **RandInt**(command to generate a list of 50 values: 0s will represent tails and 1s will represent heads.

Think about a formula you could use to quickly calculate the total number of heads from those 50 flips.

Enter the formula for the sum in cell C1.

Record the calculated value in cell D1.

Now we need to repeat the experiment a few more times. Select Column B and choose **MENU > Actions > Recalculate** to randomly repopulate the column with 1s and 0s.

Repeat for a total of 10 trials, keeping a record of each sum by entering it in the next empty cell in Column D.

Select Column D and make a **Quick Graph** of the data. Adjust the **Window Settings**: set the Xmin and Xmax at 0 and 50, respectively.

You should find that the data points cluster together around the "middle," or about 25. If they do not appear this way, it might be that not enough trials of the experiment were performed for a pattern to appear. We will explore even more trials in the next problem.

Now suppose instead of flipping the coin 50 times, you flip it 500 times. And instead of performing 10 trials, you perform 50 trials! How many heads would you expect to flip on each trial?

On the next page, each row represents one trial, or 500 flips. Try using the formula **RandInt(0,500,50)** to generate random numbers of heads in 500 flips.

1.7		1.8		2.1		2.2		RAD AUTO REAL			
A		trial		B		numheads		C		numtails	
1		1									
2		2									
3		3									
4		4									
5		5									
6		6									
AI											

Go back and look closely at your data.

Of the 50 values, how many are under 200?

Can you explain why this doesn't make sense?

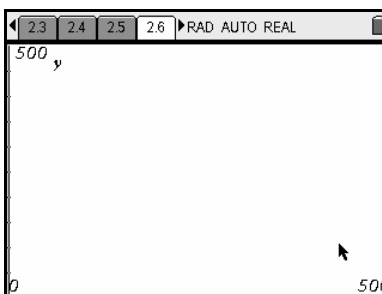
The function $f(x)$ has been defined so as to *correctly* represent a random number of heads out of 500 flips.

Return to the spreadsheet and clear the formula cell for Column B. Enter " $=f(A1)$ " in cell B1, and **Fill Down** from B1 to populate all 50 trials.

Does this data make more sense?

Based on these results, enter a formula for Column C that will give the number of *tails* for each trial. If you **Recalculate** Column B, the values in Column C should update automatically.

Display the scatter plot *numheads* v. *numtails* on page 2.6.



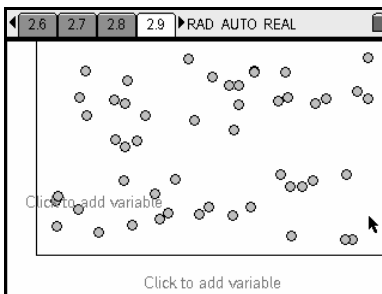
What do you notice about the graph? Do the points fill the viewing window?

Select **Zoom - Data** from the Window menu to gain a closer view of the data.

Describe this new distribution--What are the minimum and maximum number of heads? Are the points evenly spaced? Explain.

Now we'll take a different look at the data. On the next page, create a histogram of the data in Column B, **numheads**.

Adjust the window so you can see a little past each of the "bookend" bars and above the highest bar.



You can grab and drag the side of a bar to change the width of the intervals. Clicking on a bar displays its interval and the number of trials in that interval.

- Theoretically, how many heads would you expect out of 500 flips?
- What percent of these 50 trials resulted in 240-260 heads?
- What about 230-270 heads?