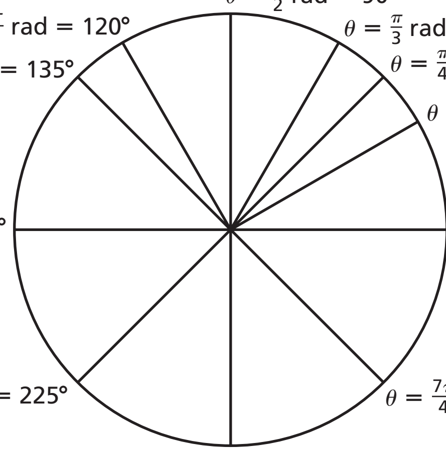
Date ____________ Period ____________ Name ____________________

ACTIVITY 7

Connecting Math to Physics

Angular Measurements

Angular measurements indicate the amount of rotation. The unit most often used is the degree, where $1° = \frac{1}{360}$ of a circle. In the figure below, θ represents the rotation, the measurement of an angle. When $\theta = 360°$, a full circle has been measured.



Another type of angular measurement is the radian. The radian measurement is equal to the arc length, d, of a circle with a unit radius (radius of 1). An arc is any portion of the circle's perimeter, or circumference. The figure above shows the angular measurements of a circle in both degrees and radians.

How is the length of an arc measured? Recall that the circumference of a circle is $2\pi r$. If $r = 1$, $C = 2\pi$. For a full revolution, $\theta = 2\pi$ rad $= 360°$. For half of a revolution, $\theta = \frac{C}{2}\text{rad} = \frac{2\pi}{2}\text{rad} = \pi$ rad $= 180°$. To convert between rads and degrees, use the equality π rad $= 180°$ to form a conversion factor.

Convert from radians to degrees.

1. $\frac{3\pi}{2}$ rad

2. $\frac{\pi}{6}$ rad

3. $\frac{\pi}{8}$ rad

4. $\frac{3\pi}{4}$ rad

Convert from degrees to radians. Use π in your answers. (Hint: use the conversion factor to create a fraction, and then reduce the fraction.)

5. 360°

6. 45°

7. 135°

8. 60°

When the radius is not equal to one, the angular measurement in rads is not the same as the arc length, d. The relationship between d, r, and θ is as follows: $d = r\theta$, where θ is in radians (not degrees). When $r = 1$, $d = \theta$.

Find the missing value. (Note: d refers to the arc length, not the diameter.)

9. When $\theta = \pi$ rad and $r = 2.0$ m, find d.

10. When $\theta = \frac{\pi}{3}$ rad and $r = 9.0$ mm, find d.

11. When $\theta = 2\pi$ rad and $d = 2.0$ cm, find r.

12. When $\theta = \frac{2\pi}{3}$ rad and $d = 10.0$ m, find r.

13. When $d = 10.0$ mm and $r = 2.0$ mm, find θ.

14. When $d = 90.0$ km and $r = 30.0$ km, find θ.

Date ______ Period ______ Name ______

ACTIVITY 9

Connecting Math to Physics

Exponential Notation

In the equation for kinetic energy, $KE = \frac{1}{2}mv^2$, the variable v has an exponent of two. The exponent is the small number to the upper right of the variable. The general form of exponential notation is b^x, where b is the base and x is the exponent. Exponential notation is applicable to many situations where it is necessary to multiply a number or variable by itself several times. Rather than writing the base multiple times, it is written once with an exponent. The exponent is equal to the number of times the base is multiplied by itself. For example, $v \times v = v^2$ and $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 8^6$. If a number or a variable has an exponent of one, the exponent is generally omitted: $m^1 = m$ and $3^1 = 3$. In the equation $a = \frac{d}{t^2}$, the exponent is in the denominator. This equation may be rewritten as $a = dt^{-2}$, where t has a negative exponent. The negative exponent rule states that $\frac{1}{b^x} = b^{-x}$. For example, $\frac{1}{9^3} = 9^{-3}$ and $\frac{1}{6^{-7}} = 6^{-(-7)} = 6^7$. An exponent also may be a fraction. Square and cubed roots are examples of numbers written with exponents as fractions. The square root sign, $\sqrt{b}$, may be written as $b^{\frac{1}{2}}$ and the cubed root $\sqrt[3]{b}$ may be written as $b^{\frac{1}{3}}$.

Rewrite the expression in exponential notation with a positive exponent.

1. $9 \times 9 \times 9$ ______

2. $2 \times 2 \times 2 \times 2$ ______

3. 1 ______

4. $a \times a$ ______

5. 7^{-5} ______

6. v^{-8} ______

7. $\frac{1}{3^{-4}}$ ______

8. $\frac{1}{z^{-x}}$ ______

9. $\sqrt{16}$ ______

10. $\sqrt{c^2}$ ______

11. $\sqrt[3]{8}$ ______

12. $\sqrt[3]{x^3}$ ______

Exponent product rule: When multiplying two quantities with the same base, the product may be written as the base with the exponents added together: $b^x \times b^y = b^{x+y}$. For example, $3^2 \times 3^3 = 3^{2+3} = 3^5 = 243$. The product rule may be illustrated by expanding the exponents: $3^2 \times 3^3 = (3 \times 3) \times (3 \times 3 \times 3) = 3^5 = 243$.

Exponent quotient rule: When dividing two quantities with the same base, subtract the exponents: $\frac{b^x}{b^y} = b^{x-y}$. For example, $\frac{4^3}{4^5} = 4^{3-5} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$. The quotient rule may be illustrated by expanding the exponents:

$$\frac{4^3}{4^5} = \frac{4 \times 4 \times 4}{4 \times 4 \times 4 \times 4 \times 4} = \frac{1}{4 \times 4} = \frac{1}{4^2} = \frac{1}{16}.$$

Find the decimal value of the expression.

13. 10^4 ________

14. 2.00^{-3} ________

15. $8.00^1 \times 8.00^2$ ________

16. $5.0^{-4} \times 5.0^6$ ________

17. $7.0^{-6} \times 7.0^5$ ________

18. $9.0^{-1} \times 9.0^{-1}$ ________

19. $\frac{6.0^4}{6.0^2}$ ________

20. $\frac{3^9}{3^8}$ ________

21. $\frac{2.0^2}{2.0^{-4}}$ ________

22. $\frac{4^{-2}}{4^{-3}}$ ________

23. $\frac{5^{-2}}{5^{-1}}$ ________

The exponent product and quotient rules are applicable to numbers written in scientific notation. Numbers in scientific notation have a number multiplied by an exponent with a base of 10. For example, 4.0×10^3 and 2.1×10^{-2} are numbers written in scientific notation. To find the product of these numbers, multiply the standard number portions and the base 10 portions separately. To multiply the base 10 portions, use the exponent product rule. $(3.0\times10^3) \times (2.0\times10^2) = (3.0 \times 2.0) \times (10^3 \times 10^2) = 6.0\times10^{3+2} = 6.0\times10^5$. This same procedure may be used to divide the numbers in scientific notation, only the decimal numbers are divided and the exponent quotient rule is used.

$$(4.0\times10^3) \div (2.1\times10^2) = \frac{4.0\times10^3}{2.1\times10^2} = \left(\frac{4.0}{2.1}\right) \times \left(\frac{10^3}{10^2}\right) = 1.9\times10^{3-2} = 1.9\times10^1$$

Find the decimal value of the expression.

24. $(2.0\times10^4) \times (3.0\times10^5)$

25. $(1.0\times10^6) \times (5.0\times10^{-3})$

26. $(4.2\times10^{-2}) \times (1.0\times10^{-3})$

27. $\frac{6.0\times10^3}{2.0\times10^2}$

28. $\frac{9.9\times10^4}{3.0\times10^{-2}}$