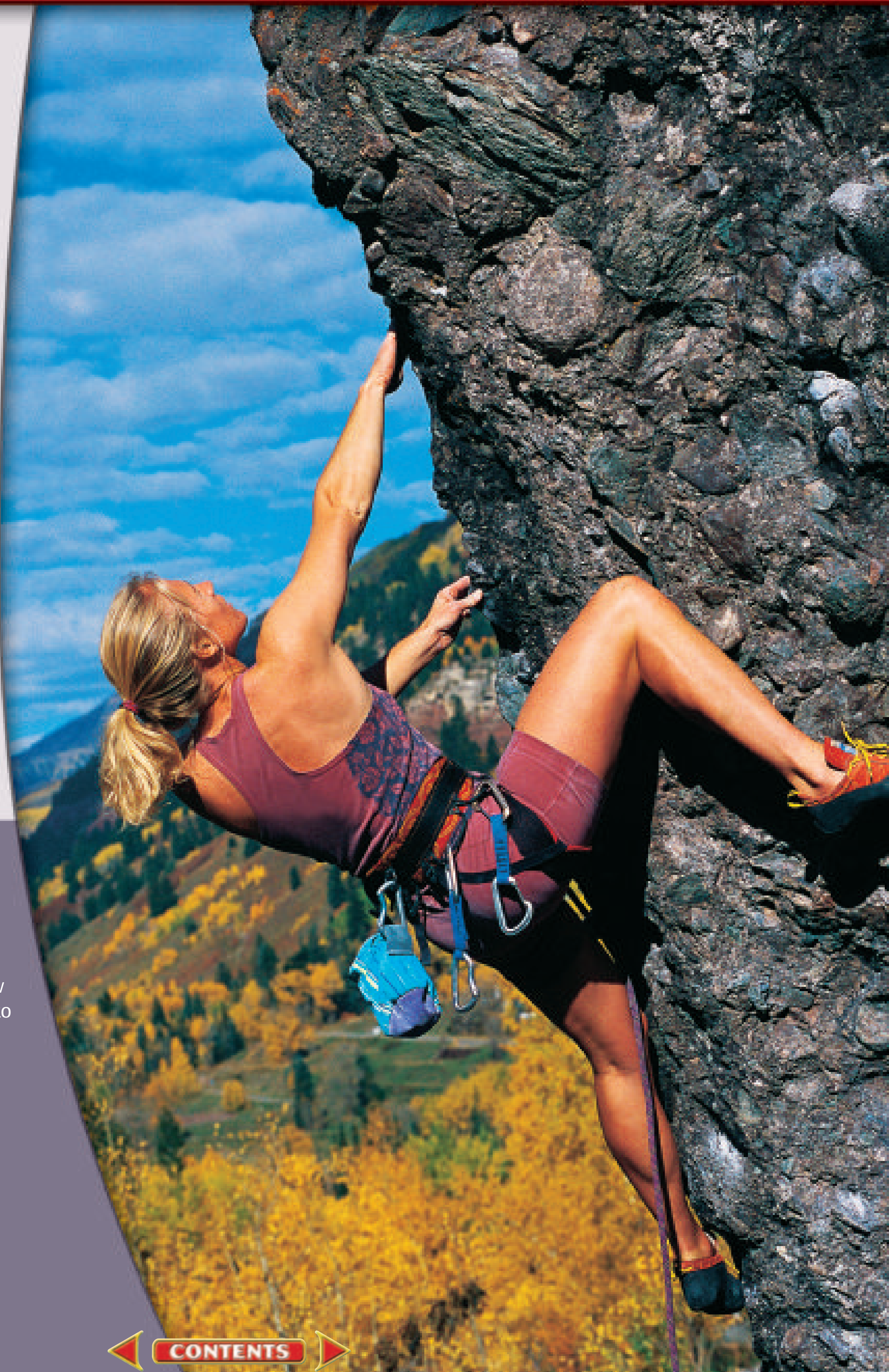
Chapter 5

Forces in Two Dimensions



What You'll Learn

- You will represent vector quantities both graphically and algebraically.
- You will use Newton's laws to analyze motion when friction is involved.
- You will use Newton's laws and your knowledge of vectors to analyze motion in two dimensions.

Why It's Important

Most objects experience forces in more than one dimension. A car being towed, for example, experiences upward and forward forces from the tow truck and the downward force of gravity.

Rock Climbing How do rock climbers keep from falling? This climber has more than one support point, and there are multiple forces acting on her in multiple directions.

Think About This ▶

A rock climber approaches a portion of the rock face that forces her to hang with her back to the ground. How will she use her equipment to apply the laws of physics in her favor and overcome this obstacle?

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Can 2 N + 2 N = 2 N?

Question

Under what conditions can two different forces equal one other force?

Procedure

1. **Measure** Use a spring scale to measure and record the weight of a 200-g object.
2. Obtain another spring scale, and attach one end of a 35-cm-long piece of string to the hooks on the bottom of each spring scale.
3. Tie one end of a 15-cm-long piece of string to the 200-g object. Loop the other end over the 35-cm-long piece of string and tie the end to the 200-g object. ***CAUTION: Avoid falling masses.***
4. Hold the spring scales parallel to each other so that the string between them forms a 120° angle. Move the string with the hanging object until both scales have the same reading. Record the readings on each scale.
5. **Collect and Organize Data** Slowly pull the string more and more horizontal while it is still supporting the 200-g object. Describe your observations.

Analysis

Does the sum of the forces measured by the two spring scales equal the weight of the hanging object? Is the sum greater than the weight? Less than the weight?

Critical Thinking Draw an equilateral triangle, with one side vertical, on a sheet of paper. If the two sides of the triangle are 2.0 N, explain the size of the third side. How is it possible that 2 N + 2 N = 2 N?

5.1 Vectors

How do rock climbers keep from falling in situations like the one shown on the preceding page? Notice that the climber has more than one support point and that there are multiple forces acting on her. She tightly grips crevices in the rock and has her feet planted on the rock face, so there are two contact forces acting on her. Gravity is pulling on her as well, so there are three total forces acting on the climber. One aspect of this situation that is different from the ones that you have studied in earlier chapters is that the forces exerted by the rock face on the climber are not horizontal or vertical forces. You know from previous chapters that you can pick your coordinate system and orient it in the way that is most useful to analyzing the situation. But what happens when the forces are not at right angles to each other? How can you set up a coordinate system and find for a net force when you are dealing with more than one dimension?

▶ Objectives

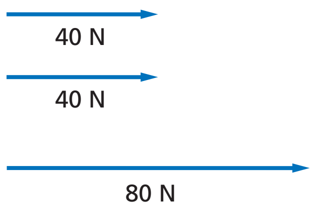
- **Evaluate** the sum of two or more vectors in two dimensions graphically.
- **Determine** the components of vectors.
- **Solve** for the sum of two or more vectors algebraically by adding the components of the vectors.

▶ Vocabulary

components
vector resolution

Vectors Revisited

Consider an example with force vectors. Recall the case in Chapter 4 in which you and a friend both pushed on a table together. Suppose that you each exerted 40 N of force to the right. **Figure 5-1** represents these vectors in a free-body diagram with the resultant vector, the net force, shown below it. The net force vector is 80 N, which is what you probably expected. But how was this net force vector obtained?

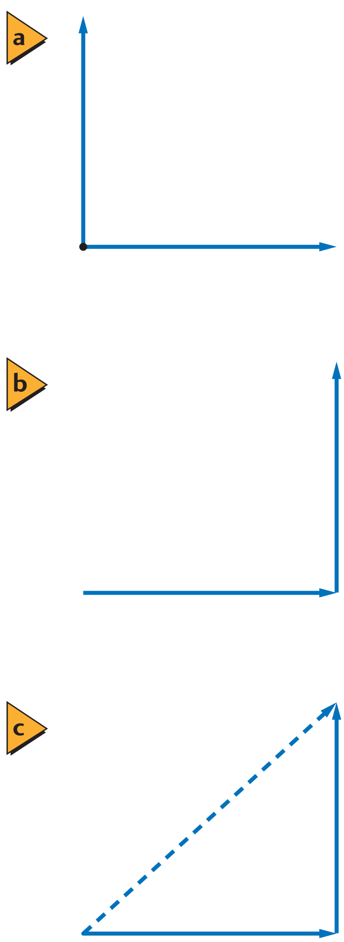


■ **Figure 5-1** The sum of the two 40-N forces is shown by the resultant vector below them.

Vectors in Multiple Dimensions

The process for adding vectors works even when the vectors do not point along the same straight line. If you are solving one of these two-dimensional problems graphically, you will need to use a protractor, both to draw the vectors at the correct angles and also to measure the direction and magnitude of the resultant vector. You can add vectors by placing them tip-to-tail and then drawing the resultant of the vector by connecting the tail of the first vector to the tip of the second vector, as shown in **Figure 5-2**. **Figure 5-2a** shows the two forces in the free-body diagram. In **Figure 5-2b,** one of the vectors has been moved so that its tail is at the same place as the tip of the other vector. Notice that its length and direction have not changed. Because the length and direction are the only important characteristics of the vector, the vector is unchanged by this movement. This is always true: if you move a vector so that its length and direction are unchanged, the vector is unchanged. Now, as in **Figure 5-2c,** you can draw the resultant vector pointing from the tail of the first vector to the tip of the last vector and measure it to obtain its magnitude. Use a protractor to measure the direction of the resultant vector. Sometimes you will need to use trigonometry to determine the length or direction of resultant vectors. Remember that the length of the hypotenuse of a right triangle can be found by using the Pythagorean theorem. If you were adding together two vectors at right angles, vector ***A*** pointing north and vector ***B*** pointing east, you could use the Pythagorean theorem to find the magnitude of the resultant, R.

■ **Figure 5-2** Add vectors by placing them tip-to-tail and drawing the resultant from the tail of the first vector to the tip of the last vector.



Pythagorean Theorem $R^2 = A^2 + B^2$

If vector A is at a right angle to vector B, then the sum of the squares of the magnitudes is equal to the square of the magnitude of the resultant vector.

If the two vectors to be added are at an angle other than 90°, then you can use the law of cosines or the law of sines.

Law of Cosines $R^2 = A^2 + B^2 - 2AB \cos \theta$

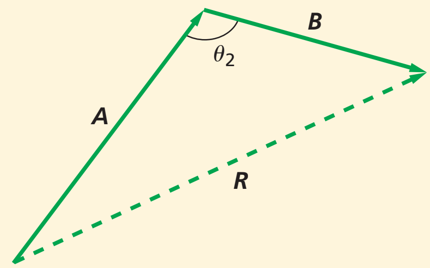
The square of the magnitude of the resultant vector is equal to the sum of the magnitudes of the squares of the two vectors, minus two times the product of the magnitudes of the vectors, multiplied by the cosine of the angle between them.

Law of Sines $\dfrac{R}{\sin \theta} = \dfrac{A}{\sin a} = \dfrac{B}{\sin b}$

The magnitude of the resultant, divided by the sine of the angle between two vectors, is equal to the magnitude of one of the vectors divided by the angle between that component vector and the resultant vector.

EXAMPLE Problem 1

Finding the Magnitude of the Sum of Two Vectors **Find the magnitude of the sum of a 15-km displacement and a 25-km displacement when the angle between them is 90° and when the angle between them is 135°.**



1 Analyze and Sketch the Problem

- Sketch the two displacement vectors, ***A*** and ***B***, and the angle between them.

Known:		Unknown:
$A = 25$ km	$\theta_1 = 90°$	$R = ?$
$B = 15$ km	$\theta_2 = 135°$	

2 Solve for the Unknown

When the angle is 90°, use the Pythagorean theorem to find the magnitude of the resultant vector.

$$R^2 = A^2 + B^2$$
$$R = \sqrt{A^2 + B^2}$$
$$= \sqrt{(25\text{ km})^2 + (15\text{ km})^2}$$ **Substitute *A* = 25 km, *B* = 15 km**
$$= 29\text{ km}$$

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Square and Cube Roots
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When the angle does not equal 90°, use the law of cosines to find the magnitude of the resultant vector.

$$R^2 = A^2 + B^2 - 2AB(\cos\theta_2)$$
$$R = \sqrt{A^2 + B^2 - 2AB(\cos\theta_2)}$$
$$= \sqrt{(25\text{ km})^2 + (15\text{ km})^2 - 2(25\text{ km})(15\text{ km})(\cos 135°)}$$ **Substitute *A* = 25 km, *B* = 15 km, θ_2 = 135°**
$$= 37\text{ km}$$

3 Evaluate the Answer

- **Are the units correct?** Each answer is a length measured in kilometers.
- **Do the signs make sense?** The sums are positive.
- **Are the magnitudes realistic?** The magnitudes are in the same range as the two combined vectors, but longer. This is because each resultant is the side opposite an obtuse angle. The second answer is larger than the first, which agrees with the graphical representation.

PRACTICE Problems

Additional Problems, Appendix B

1. A car is driven 125.0 km due west, then 65.0 km due south. What is the magnitude of its displacement? Solve this problem both graphically and mathematically, and check your answers against each other.
2. Two shoppers walk from the door of the mall to their car, which is 250.0 m down a lane of cars, and then turn 90° to the right and walk an additional 60.0 m. What is the magnitude of the displacement of the shoppers' car from the mall door? Solve this problem both graphically and mathematically, and check your answers against each other.
3. A hiker walks 4.5 km in one direction, then makes a 45° turn to the right and walks another 6.4 km. What is the magnitude of her displacement?
4. An ant is crawling on the sidewalk. At one moment, it is moving south a distance of 5.0 mm. It then turns southwest and crawls 4.0 mm. What is the magnitude of the ant's displacement?

Components of Vectors

Choosing a coordinate system, such as the one in **Figure 5-3a,** is similar to laying a grid drawn on a sheet of transparent plastic on top of a vector problem. You have to choose where to put the center of the grid (the origin) and establish the directions in which the axes point. Notice that in the coordinate system shown in Figure 5-3a, the *x*-axis is drawn through the origin with an arrow pointing in the positive direction. The positive *y*-axis is located 90° counterclockwise from the positive *x*-axis and crosses the *x*-axis at the origin.

How do you choose the direction of the *x*-axis? There is never a single correct answer, but some choices make the problem easier to solve than others. When the motion you are describing is confined to the surface of Earth, it is often convenient to have the *x*-axis point east and the *y*-axis point north. When the motion involves an object moving through the air, the positive *x*-axis is often chosen to be horizontal and the positive *y*-axis vertical (upward). If the motion is on a hill, it's convenient to place the positive *x*-axis in the direction of the motion and the *y*-axis perpendicular to the *x*-axis.

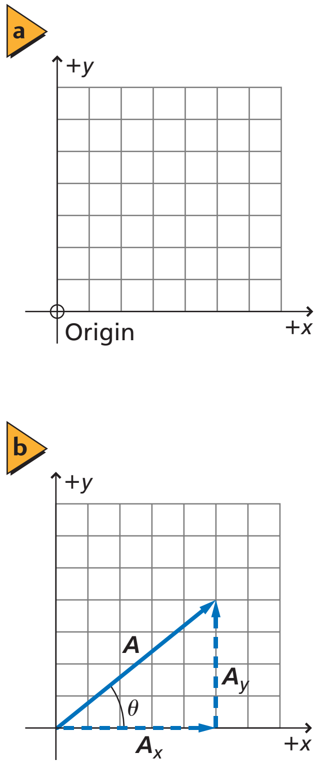


Figure 5-3 A coordinate system has an origin and two perpendicular axes **(a)**. The direction of a vector, ***A***, is measured counterclockwise from the *x*-axis **(b).**

Component vectors Defining a coordinate system allows you to describe a vector in a different way. Vector ***A*** shown in **Figure 5-3b,** for example, could be described as going 5 units in the positive *x*-direction and 4 units in the positive *y*-direction. You can represent this information in the form of two vectors like the ones labeled $\boldsymbol{A}_x$ and $\boldsymbol{A}_y$ in the diagram. Notice that $\boldsymbol{A}_x$ is parallel to the *x*-axis, and $\boldsymbol{A}_y$ is parallel to the *y*-axis. Further, you can see that if you add $\boldsymbol{A}_x$ and $\boldsymbol{A}_y$, the resultant is the original vector, ***A***. A vector can be broken into its **components,** which are a vector parallel to the *x*-axis and another parallel to the *y*-axis. This can always be done and the following vector equation is always true.

$$\boldsymbol{A} = \boldsymbol{A}_x + \boldsymbol{A}_y$$

This process of breaking a vector into its components is sometimes called **vector resolution.** Notice that the original vector is the hypotenuse of a right triangle. This means that the magnitude of the original vector will always be larger than the magnitudes of either component vector.

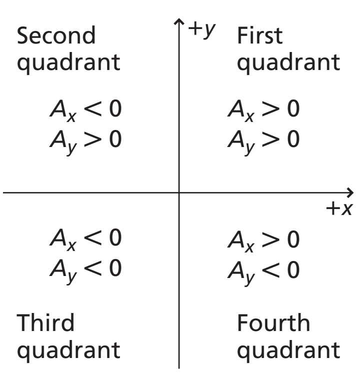
Another reason for choosing a coordinate system is that the direction of any vector can be specified relative to those coordinates. The direction of a vector is defined as the angle that the vector makes with the *x*-axis, measured counterclockwise. In Figure 5-3b, the angle, θ, tells the direction of the vector, ***A***. All algebraic calculations involve only the positive components of vectors, not the vectors themselves. In addition to measuring the lengths of the component vectors graphically, you can find the components by using trigonometry. The components are calculated using the equations below, where the angle, θ, is measured counterclockwise from the positive *x*-axis.

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A_x}{A}; \text{ therefore, } A_x = A\cos\theta$$

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{A_y}{A}; \text{ therefore, } A_y = A\sin\theta$$

When the angle that a vector makes with the *x*-axis is larger than 90°, the sign of one or more components is negative, as shown in **Figure 5-4.**

Figure 5-4 The sign of a component depends upon which of the four quadrants the component is in.



Algebraic Addition of Vectors

You might be wondering why you need to resolve vectors into their components. The answer is that doing this often makes adding vectors together much easier mathematically. Two or more vectors (***A***, ***B***, ***C***, etc.) may be added by first resolving each vector into its *x*- and *y*-components. The *x*-components are added to form the *x*-component of the resultant: $R_x = A_x + B_x + C_x$. Similarly, the *y*-components are added to form the *y*-component of the resultant: $R_y = A_y + B_y + C_y$. This process is illustrated graphically in **Figure 5-5.** Because R_x and R_y are at a right angle (90°), the magnitude of the resultant vector can be calculated using the Pythagorean theorem, $R^2 = R_x^2 + R_y^2$. To find the angle or direction of the resultant, recall that the tangent of the angle that the vector makes with the *x*-axis is given by the following.

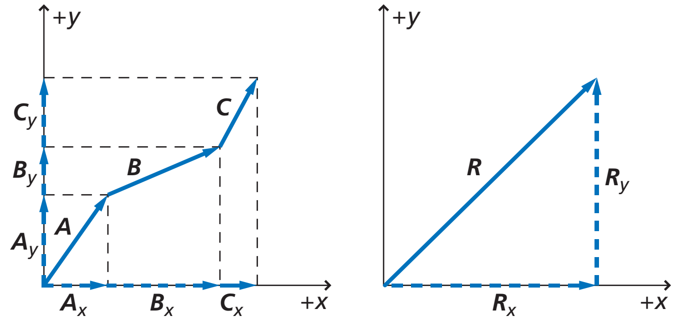


Figure 5-5 ***R***$_x$ is the sum of the *x*-components of ***A***, ***B***, and ***C***. ***R***$_y$ is the sum of the *y*-components. The vector sum of ***R***$_x$ and ***R***$_y$ is the vector sum of ***A***, ***B***, and ***C***.

Angle of the Resultant Vector $\quad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$

The angle of the resultant vector is equal to the inverse tangent of the quotient of the *y*-component divided by the *x*-component of the resultant vector.

You can find the angle by using the $\tan^{-1}$ key on your calculator. Note that when $\tan\theta > 0$, most calculators give the angle between 0° and 90°, and when $\tan\theta < 0$, the angle is reported to be between 0° and −90°.

PROBLEM-SOLVING Strategies

Vector Addition

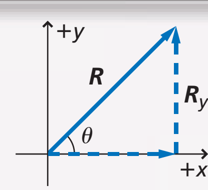
Use the following technique to solve problems for which you need to add or subtract vectors.

1. Choose a coordinate system.
2. Resolve the vectors into their *x*-components using $A_x = A\cos\theta$, and their *y*-components using $A_y = A\sin\theta$, where θ is the angle measured counterclockwise from the positive *x*-axis.
3. Add or subtract the component vectors in the *x*-direction.
4. Add or subtract the component vectors in the *y*-direction.
5. Use the Pythagorean theorem, $R = \sqrt{R_x^2 + R_y^2}$, to find the magnitude of the resultant vector.
6. Use $\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$ to find the angle of the resultant vector.

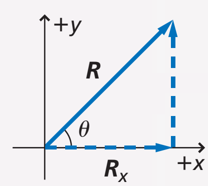
Connecting Math to Physics

Math Review

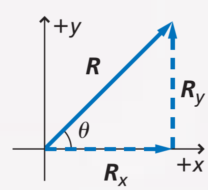
$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{R_y}{R}$$



$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{R_x}{R}$$



$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{R_y}{R_x}$$

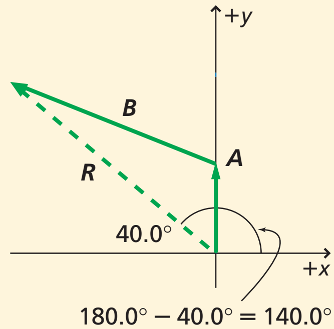


▶ EXAMPLE Problem 2

Finding Your Way Home A GPS receiver indicates that your home is 15.0 km and 40.0° north of west, but the only path through the woods leads directly north. If you follow the path 5.0 km before it opens into a field, how far, and in what direction, would you have to walk to reach your home?

1 Analyze and Sketch the Problem

- Draw the resultant vector, $\boldsymbol{R}$, from your original location to your home.
- Draw $\boldsymbol{A}$, the known vector, and draw $\boldsymbol{B}$, the unknown vector.



Known:

$\boldsymbol{A}$ = 5.0 km, due north

$\boldsymbol{R}$ = 15.0 km, 40.0° north of west

θ = 140.0°

Unknown:

$\boldsymbol{B}$ = ?

2 Solve for the Unknown

Find the components of $\boldsymbol{R}$.

$$R_x = R\cos\theta$$
$$= (15.0 \text{ km})\cos 140.0° \quad \text{Substitute } \boldsymbol{R} = 15.0 \text{ km}, \theta = 140.0°$$
$$= -11.5 \text{ km}$$

$$R_y = R\sin\theta$$
$$= (15.0 \text{ km})\sin 140.0° \quad \text{Substitute } \boldsymbol{R} = 15.0 \text{ km}, \theta = 140.0°$$
$$= 9.64 \text{ km}$$

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Inverses of Sine, Cosine, and Tangent
page 856

Because $\boldsymbol{A}$ is due north, A_x = 0.0 km and A_y = 5.0 km.

Use the components of $\boldsymbol{R}$ and $\boldsymbol{A}$ to find the components of $\boldsymbol{B}$.

$$B_x = R_x - A_x$$
$$= -11.5 \text{ km} - 0.0 \text{ km} \quad \text{Substitute } R_x = -11.5 \text{ km}, A_x = 0.0 \text{ km}$$
$$= -11.5 \text{ km} \quad \text{The negative sign means that this component points west.}$$

$$B_y = R_y - A_y$$
$$= 9.64 \text{ km} - 5.0 \text{ km} \quad \text{Substitute } R_y = 9.64 \text{ km}, A_y = 5.0 \text{ km}$$
$$= 4.6 \text{ km} \quad \text{This component points north.}$$

Use the components of vector $\boldsymbol{B}$ to find the magnitude of vector $\boldsymbol{B}$.

$$B = \sqrt{B_x^{\,2} + B_y^{\,2}}$$
$$= \sqrt{(-11.5 \text{ km})^2 + (4.6 \text{ km})^2} \quad \text{Substitute } B_y = -4.6 \text{ km}, B_x = -11.5 \text{ km}$$
$$= 12 \text{ km}$$

Use the tangent to find the direction of vector $\boldsymbol{B}$.

$$\theta = \tan^{-1}\frac{B_y}{B_x}$$
$$= \tan^{-1}\frac{4.6 \text{ km}}{-11.5 \text{ km}} \quad \text{Substitute } B_y = 4.6 \text{ km}, B_x = -11.5 \text{ km}$$
$$= -22° \text{ or } 158° \quad \text{Tangent of an angle is negative in quadrants II and IV, so two answers are possible.}$$

Locate the tail of vector $\boldsymbol{B}$ at the origin of a coordinate system and draw the components B_x and B_y. The direction is in the third quadrant, at 158°, or 22° north of west. Thus, $\boldsymbol{B}$ = 12 km at 22° north of west.

3 Evaluate the Answer

- **Are the units correct?** Kilometers and degrees are correct.
- **Do the signs make sense?** They agree with the diagram.
- **Is the magnitude realistic?** The length of $\boldsymbol{B}$ should be longer than R_x because the angle between A and B is greater than 90°.

PRACTICE Problems

Additional Problems, Appendix B

Solve problems 5–10 algebraically. You may also choose to solve some of them graphically to check your answers.

5. Sudhir walks 0.40 km in a direction 60.0° west of north, then goes 0.50 km due west. What is his displacement?

6. Afua and Chrissy are going to sleep overnight in their tree house and are using some ropes to pull up a box containing their pillows and blankets, which have a total mass of 3.20 kg. The girls stand on different branches, as shown in **Figure 5-6,** and pull at the angles and with the forces indicated. Find the *x*- and *y*-components of the net force on the box. *Hint: Draw a free-body diagram so that you do not leave out a force.*

7. You first walk 8.0 km north from home, then walk east until your displacement from home is 10.0 km. How far east did you walk?

8. A child's swing is held up by two ropes tied to a tree branch that hangs 13.0° from the vertical. If the tension in each rope is 2.28 N, what is the combined force (magnitude and direction) of the two ropes on the swing?

9. Could a vector ever be shorter than one of its components? Equal in length to one of its components? Explain.

10. In a coordinate system in which the *x*-axis is east, for what range of angles is the *x*-component positive? For what range is it negative?

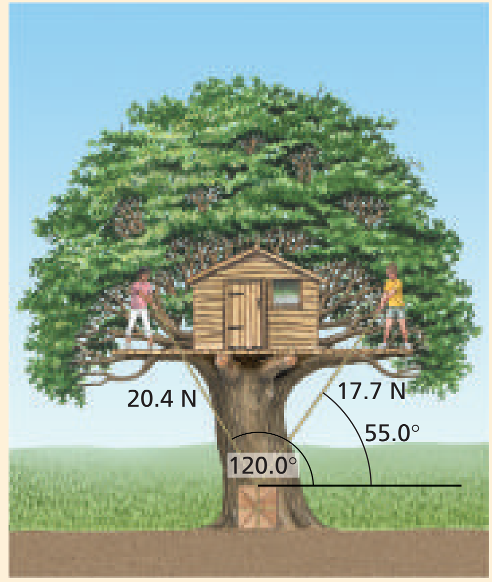


Figure 5-6
(Not to scale)

You will use these techniques to resolve vectors into their components throughout your study of physics. You will get more practice at it, particularly in the rest of this chapter and the next. Resolving vectors into components allows you to analyze complex systems of vectors without using graphical methods.

5.1 Section Review

11. Distance v. Displacement Is the distance that you walk equal to the magnitude of your displacement? Give an example that supports your conclusion.

12. Vector Difference Subtract vector ***K*** from vector ***L***, shown in **Figure 5-7.**

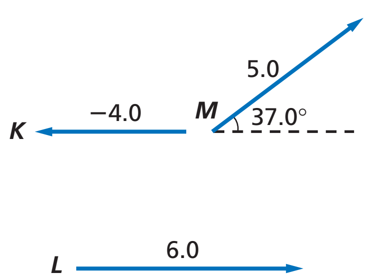


Figure 5-7

13. Components Find the components of vector ***M***, shown in Figure 5-7.

14. Vector Sum Find the sum of the three vectors shown in Figure 5-7.

15. Commutative Operations The order in which vectors are added does not matter. Mathematicians say that vector addition is commutative. Which ordinary arithmetic operations are commutative? Which are not?

16. Critical Thinking A box is moved through one displacement and then through a second displacement. The magnitudes of the two displacements are unequal. Could the displacements have directions such that the resultant displacement is zero? Suppose the box was moved through three displacements of unequal magnitude. Could the resultant displacement be zero? Support your conclusion with a diagram.

5.2 Friction

▶ **Objectives**

- **Define** the friction force.
- **Distinguish** between static and kinetic friction.

▶ **Vocabulary**

kinetic friction
static friction
coefficient of kinetic friction
coefficient of static friction

Push your hand across your desktop and feel the force called friction opposing the motion. Push your book across the desk. When you stop pushing, the book will continue moving for a little while, then it will slow down and stop. The frictional force acting on the book gave it an acceleration in the direction opposite to the one in which it was moving. So far, you have neglected friction in solving problems, but friction is all around you. You need it to both start and stop a bicycle and a car. If you have ever walked on ice, you understand the importance of friction.

Static and Kinetic Friction

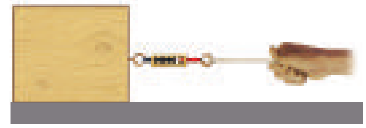
There are two types of friction. Both always oppose motion. When you pushed your book across the desk, it experienced a type of friction that acts on moving objects. This force is known as **kinetic friction,** and it is exerted on one surface by another when the two surfaces rub against each other because one or both of them are moving.

To understand the other kind of friction, imagine trying to push a heavy couch across the floor. You give it a push, but it does not move. Because it does not move, Newton's laws tell you that there must be a second horizontal force acting on the couch, one that opposes your force and is equal in size. This force is **static friction,** which is the force exerted on one surface by another when there is no motion between the two surfaces. You might push harder and harder, as shown in **Figures 5-8a** and **5-8b,** but if the couch still does not move, the force of friction must be getting larger. This is because the static friction force acts in response to other forces. Finally, when you push hard enough, as shown in **Figure 5-8c,** the couch will begin to move. Evidently, there is a limit to how large the static friction force can be. Once your force is greater than this maximum static friction, the couch begins moving and kinetic friction begins to act on it instead of static friction.

A model for friction forces On what does a frictional force depend? The materials that the surfaces are made of play a role. For example, there is more friction between skis and concrete than there is between skis and snow. It may seem reasonable to think that the force of friction also might depend on either the surface area in contact or the speed of the motion, but experiments have shown that this is not true. The normal force between the two objects does matter, however. The harder one object is pushed against the other, the greater the force of friction that results.

■ **Figure 5-8** There is a limit to the ability of the static friction force to match the applied force.

If you pull a block along a surface at a constant velocity, according to Newton's laws, the frictional force must be equal and opposite to the force with which you pull. You can pull a block of known mass along a table at a constant velocity and use a spring scale, as shown in **Figure 5-9,** to measure the force that you exert. You can then stack additional blocks on the block to increase the normal force and repeat the measurement.



■ **Figure 5-9** The spring scale pulls the block with a constant force.

Plotting the data will yield a graph like the one in **Figure 5-10.** There is a direct proportion between the kinetic friction force and the normal force. The different lines correspond to dragging the block along different surfaces. Note that the line corresponding to the sandpaper surface has a steeper slope than the line for the highly polished table. You would expect it to be much harder to pull the block along sandpaper than along a polished table, so the slope must be related to the magnitude of the resulting frictional force. The slope of this line, designated μ_k, is called the **coefficient of kinetic friction** between the two surfaces and relates the frictional force to the normal force, as shown below.

Kinetic Friction Force $F_{f,\ kinetic} = \mu_k F_N$

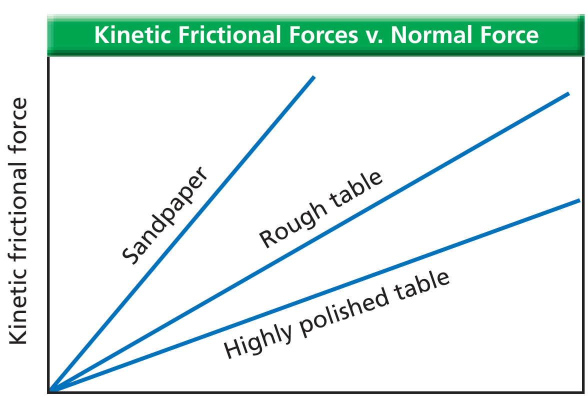
The kinetic friction force is equal to the product of the coefficient of the kinetic friction and the normal force.

The maximum static friction force is related to the normal force in a similar way as the kinetic friction force. Remember that the static friction force acts in response to a force trying to cause a stationary object to start moving. If there is no such force acting on an object, the static friction force is zero. If there is a force trying to cause motion, the static friction force will increase up to a maximum value before it is overcome and motion starts.

Static Friction Force $F_{f,\ static} \leq \mu_s F_N$

The static friction force is less than or equal to the product of the coefficient of the static friction and the normal force.

In the equation for the maximum static friction force, μ_s is the **coefficient of static friction** between the two surfaces, and $\mu_s F_N$ is the maximum static friction force that must be overcome before motion can begin. In Figure 5-8c, the static friction force is balanced the instant before the couch begins to move.



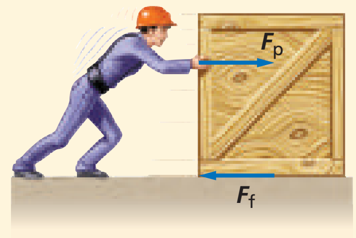
■ **Figure 5-10** There is a linear relationship between the frictional force and the normal force.

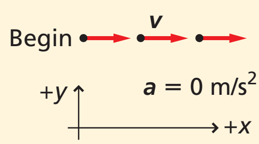
▶ EXAMPLE Problem 3

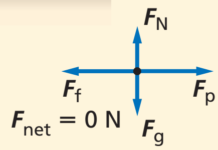
Balanced Friction Forces You push a 25.0-kg wooden box across a wooden floor at a constant speed of 1.0 m/s. How much force do you exert on the box?

1 Analyze and Sketch the Problem

- Identify the forces and establish a coordinate system.
- Draw a motion diagram indicating constant v and $a = 0$.
- Draw the free-body diagram.







Known:	Unknown:
$m = 25.0$ kg	$F_p = ?$
$v = 1.0$ m/s	
$a = 0.0$ m/s^2	
$\mu_k = 0.20$ (Table 5-1)	

2 Solve for the Unknown

The normal force is in the y-direction, and there is no acceleration.

$$F_N = F_g$$
$$= mg$$
$$= (25.0\text{ kg})(9.80\text{m/s}^2) \quad \text{Substitute } m = 25.0\text{ kg}, g = 9.80\text{ m/s}^2$$
$$= 245\text{ N}$$

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Operations with Significant Digits
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The pushing force is in the x-direction; v is constant, thus there is no acceleration.

$$F_p = \mu_k mg$$
$$= (0.20)(25.0\text{ kg})(9.80\text{ m/s}^2) \quad \text{Substitute } \mu_k = 0.20, m = 25.0\text{ kg}, g = 9.80\text{ m/s}^2$$
$$= 49\text{ N}$$

3 Evaluate the Answer

- **Are the units correct?** Performing dimensional analysis on the units verifies that force is measured in kg·m/s^2 or N.
- **Does the sign make sense?** The positive sign agrees with the sketch.
- **Is the magnitude realistic?** The force is reasonable for moving a 25.0-kg box.

▶ PRACTICE Problems

Additional Problems, Appendix B

17. A girl exerts a 36-N horizontal force as she pulls a 52-N sled across a cement sidewalk at constant speed. What is the coefficient of kinetic friction between the sidewalk and the metal sled runners? Ignore air resistance.

18. You need to move a 105-kg sofa to a different location in the room. It takes a force of 102 N to start it moving. What is the coefficient of static friction between the sofa and the carpet?

19. Mr. Ames is dragging a box full of books from his office to his car. The box and books together have a combined weight of 134 N. If the coefficient of static friction between the pavement and the box is 0.55, how hard must Mr. Ames push the box in order to start it moving?

20. Suppose that the sled in problem 17 is resting on packed snow. The coefficient of kinetic friction is now only 0.12. If a person weighing 650 N sits on the sled, what force is needed to pull the sled across the snow at constant speed?

21. Suppose that a particular machine in a factory has two steel pieces that must rub against each other at a constant speed. Before either piece of steel has been treated to reduce friction, the force necessary to get them to perform properly is 5.8 N. After the pieces have been treated with oil, what will be the required force?

Table 5-1		
Typical Coefficients of Friction		
Surface	μ_s	μ_k
Rubber on dry concrete	0.80	0.65
Rubber on wet concrete	0.60	0.40
Wood on wood	0.50	0.20
Steel on steel (dry)	0.78	0.58
Steel on steel (with oil)	0.15	0.06

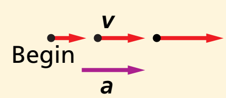
Note that the equations for the kinetic and maximum static friction forces involve only the magnitudes of the forces. The forces themselves, $\mathbf{F}_f$ and $\mathbf{F}_N$, are at right angles to each other. **Table 5-1** shows coefficients of friction between various surfaces. Although all the listed coefficients are less than 1.0, this does not mean that they must always be less than 1.0. For example, coefficients as large as 5.0 are experienced in drag racing.

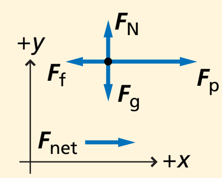
EXAMPLE Problem 4

Unbalanced Friction Forces If the force that you exert on the 25.0-kg box in Example Problem 3 is doubled, what is the resulting acceleration of the box?

1 Analyze and Sketch the Problem

- Draw a motion diagram showing v and a.
- Draw the free-body diagram with a doubled F_p.





Known:

$m = 25.0$ kg $\quad \mu_k = 0.20$

$v = 1.0$ m/s $\quad F_p = 2(49\text{ N}) = 98$ N

Unknown:

$a = ?$

2 Solve for the Unknown

The normal force is in the y-direction, and there is no acceleration.

$$F_N = F_g = mg \qquad \text{Substitute } F_g = mg$$

In the x-direction there is an acceleration. So the forces must be unequal.

$$F_{net} = F_p - F_f$$

$$ma = F_p - F_f \qquad \text{Substitute } F_{net} = ma$$

$$a = \frac{F_p - F_f}{m}$$

Find F_f and substitute it into the expression for a.

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$$F_f = \mu_k F_N$$

$$= \mu_k mg \qquad \text{Substitute } F_N = mg$$

$$a = \frac{F_p - \mu_k mg}{m} \qquad \text{Substitute } F_f = \mu_k mg$$

$$= \frac{98\text{ N} - (0.20)(25.0\text{ kg})(9.80\text{ m/s}^2)}{25.0\text{ kg}} \qquad \text{Substitute } F_p = 98\text{ N, } m = 25.0\text{ kg, } \mu_k = 0.20\text{, } g = 9.80\text{ m/s}^2$$

$$= 2.0\text{ m/s}^2$$

3 Evaluate the Answer

- **Are the units correct?** a is measured in m/s^2.
- **Does the sign make sense?** In this coordinate system, the sign should be positive.
- **Is the magnitude realistic?** If the force were cut in half, a would be zero.

▶ PRACTICE Problems

Additional Problems, Appendix B

22. A 1.4-kg block slides across a rough surface such that it slows down with an acceleration of 1.25 m/s^2. What is the coefficient of kinetic friction between the block and the surface?

23. You help your mom move a 41-kg bookcase to a different place in the living room. If you push with a force of 65 N and the bookcase accelerates at 0.12 m/s^2, what is the coefficient of kinetic friction between the bookcase and the carpet?

24. A shuffleboard disk is accelerated to a speed of 5.8 m/s and released. If the coefficient of kinetic friction between the disk and the concrete court is 0.31, how far does the disk go before it comes to a stop? The courts are 15.8 m long.

25. Consider the force pushing the box in Example Problem 4. How long would it take for the velocity of the box to double to 2.0 m/s?

26. Ke Min is driving along on a rainy night at 23 m/s when he sees a tree branch lying across the road and slams on the brakes when the branch is 60.0 m in front of him. If the coefficient of kinetic friction between the car's locked tires and the road is 0.41, will the car stop before hitting the branch? The car has a mass of 2400 kg.

APPLYING PHYSICS

▶ **Causes of Friction** All surfaces, even those that appear to be smooth, are rough at a microscopic level. If you look at a photograph of a graphite crystal magnified by a scanning tunneling microscope, the atomic level surface irregularities of the crystal are revealed. When two surfaces touch, the high points on each are in contact and temporarily bond. This is the origin of both static and kinetic friction. The details of this process are still unknown and are the subject of research in both physics and engineering. ◀

Here are a few important things to remember when dealing with frictional situations. First, friction always acts in a direction opposite to the motion (or in the case of static friction, intended motion). Second, the magnitude of the force of friction depends on the magnitude of the normal force between the two rubbing surfaces; it does not necessarily depend on the weight of either object. Finally, multiplying the coefficient of static friction and the normal force gives you the maximum static friction force. Keep these things in mind as you review this section.

5.2 Section Review

27. Friction In this section, you learned about static and kinetic friction. How are these two types of friction similar? What are the differences between static and kinetic friction?

28. Friction At a wedding reception, you notice a small boy who looks like his mass is about 25 kg running part way across the dance floor, then sliding on his knees until he stops. If the kinetic coefficient of friction between the boy's pants and the floor is 0.15, what is the frictional force acting on him as he slides?

29. Velocity Derek is playing cards with his friends, and it is his turn to deal. A card has a mass of 2.3 g, and it slides 0.35 m along the table before it stops. If the coefficient of kinetic friction between the card and the table is 0.24, what was the initial speed of the card as it left Derek's hand?

30. Force The coefficient of static friction between a 40.0-kg picnic table and the ground below it is 0.43. What is the greatest horizontal force that could be exerted on the table while it remains stationary?

31. Acceleration Ryan is moving to a new apartment and puts a dresser in the back of his pickup truck. When the truck accelerates forward, what force accelerates the dresser? Under what circumstances could the dresser slide? In which direction?

32. Critical Thinking You push a 13-kg table in the cafeteria with a horizontal force of 20 N, but it does not move. You then push it with a horizontal force of 25 N, and it accelerates at 0.26 m/s^2. What, if anything, can you conclude about the coefficients of static and kinetic friction?

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5.3 Force and Motion in Two Dimensions

You have already worked with several situations dealing with forces in two dimensions. For example, when friction acts between two surfaces, you must take into account both the frictional force that is parallel to the surface and the normal force that is perpendicular to it. So far, you have considered only the motion along a level surface. Now you will use your skill in adding vectors to analyze situations in which the forces acting on an object are at angles other than 90°.

Objectives

- **Determine** the force that produces equilibrium when three forces act on an object.
- **Analyze** the motion of an object on an inclined plane with and without friction.

Vocabulary

equilibrant

Equilibrium Revisited

Recall from Chapter 4 that when the net force on an object is zero, the object is in equilibrium. According to Newton's laws, the object will not accelerate because there is no net force acting on it; an object in equilibrium is motionless or moves with constant velocity. You have already analyzed several equilibrium situations in which two forces acted on an object. It is important to realize that equilibrium can occur no matter how many forces act on an object. As long as the resultant is zero, the net force is zero and the object is in equilibrium.

Figure 5-11a shows three forces exerted on a point object. What is the net force acting on the object? Remember that vectors may be moved if you do not change their direction (angle) or length. **Figure 5-11b** shows the addition of the three forces, ***A*, *B*,** and ***C*.** Note that the three vectors form a closed triangle. There is no net force; thus, the sum is zero and the object is in equilibrium.

Suppose that two forces are exerted on an object and the sum is not zero. How could you find a third force that, when added to the other two, would add up to zero, and therefore cause the object to be in equilibrium? To find this force, first find the sum of the two forces already being exerted on the object. This single force that produces the same effect as the two individual forces added together is called the resultant force. The force that you need to find is one with the same magnitude as the resultant force, but in the opposite direction. A force that puts an object in equilibrium is called the **equilibrant. Figure 5-12** illustrates the procedure for finding this force for two vectors. Note that this general procedure works for any number of vectors.

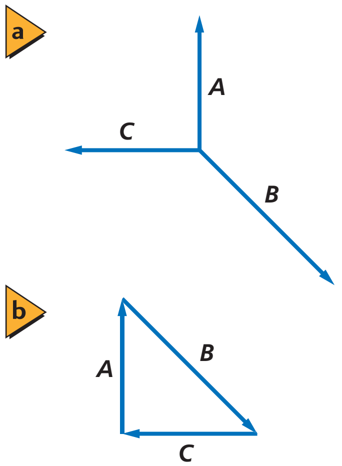
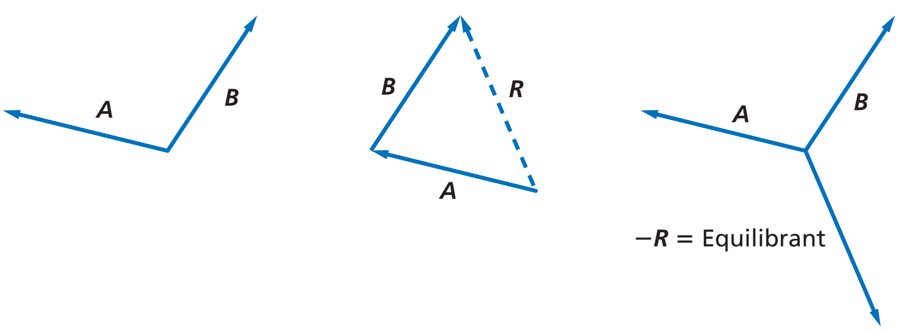


Figure 5-11 An object is in equilibrium when all the forces on it add up to zero.

Figure 5-12 The equilibrant is the same magnitude as the resultant, but opposite in direction.



CHALLENGE PROBLEM

Find the equilibrant for the following forces.

F_1 = 61.0 N at 17.0° north of east

F_2 = 38.0 N at 64.0° north of east

F_3 = 54.0 N at 8.0° west of north

F_4 = 93.0 N at 53.0° west of north

F_5 = 65.0 N at 21.0° south of west

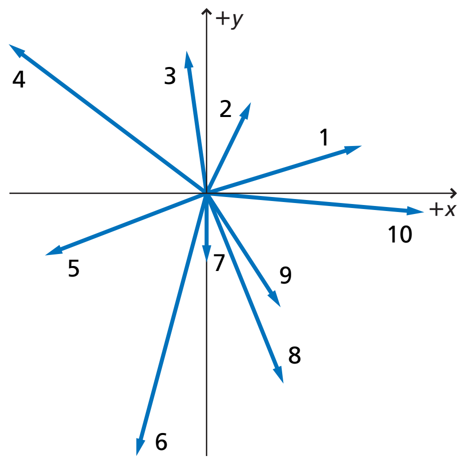
F_6 = 102.0 N at 15.0° west of south

F_7 = 26.0 N south

F_8 = 77.0 N at 22.0° east of south

F_9 = 51.0 N at 33.0° east of south

F_{10} = 82.0 N at 5.0° south of east

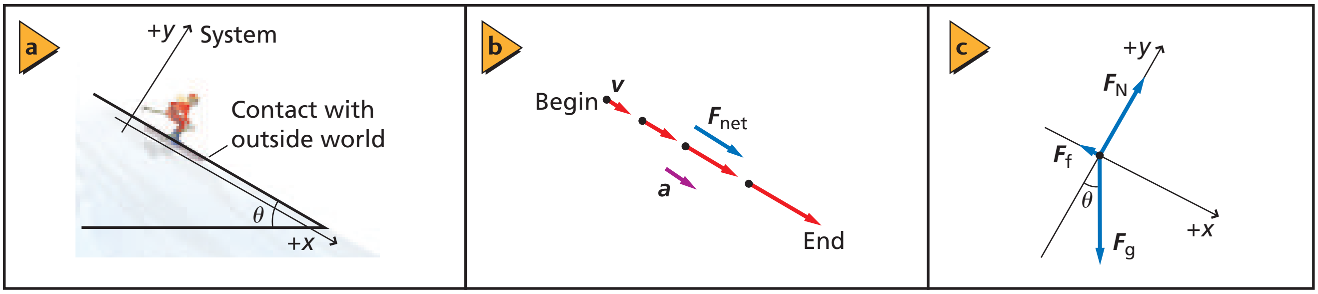


Motion Along an Inclined Plane

You have applied Newton's laws to a variety of equilibrium situations, but only to motions that were either horizontal or vertical. How would you apply them in a situation like the one in **Figure 5-13a,** in which a skier glides down a slope?

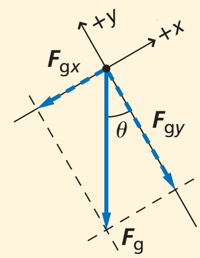
Start by identifying the forces acting on the object, the skier, as shown in **Figure 5-13b** and sketching a free-body diagram. The gravitational force on the skier is in the downward direction toward the center of Earth. There is a normal force perpendicular to the hill, and the frictional forces opposing the skier's motion are parallel to the hill. The resulting free-body diagram is shown in **Figure 5-13c.** You can see that, other than the force of friction, only one force acts horizontally or vertically, and you know from experience that the acceleration of the skier will be along the slope. How do you find the net force that causes the skier to accelerate?

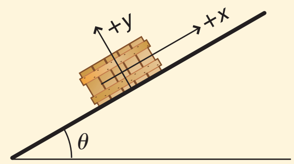
Figure 5-13 A skier slides down a slope **(a).** Identify the forces that are acting upon the skier **(b)** and draw a free-body diagram describing those forces **(c).** It is important to draw the direction of the normal and the friction forces correctly in order to properly analyze these types of situations.



▶ EXAMPLE Problem 5

Components of Weight for an Object on an Incline A crate weighing 562 N is resting on a plane inclined 30.0° above the horizontal. Find the components of the weight forces that are parallel and perpendicular to the plane.





1 Analyze and Sketch the Problem

- Include a coordinate system with the positive x-axis pointing uphill.
- Draw the free-body diagram showing $\mathbf{F}_g$, the components $\mathbf{F}_{gx}$ and $\mathbf{F}_{gy}$, and the angle θ.

Known:	**Unknown:**
$F_g = 562$ N	$F_{gx} = ?$
$\theta = 30.0°$	$F_{gy} = ?$

2 Solve for the Unknown

$\mathbf{F}_{gx}$ and $\mathbf{F}_{gy}$ are negative because they point in directions opposite to the positive axes.

$$F_{gx} = -F_g(\sin\theta)$$
$$= -(562\text{ N})(\sin 30.0°) \qquad \text{Substitute } F_g = 562,\ \theta = 30.0°$$
$$= -281\text{ N}$$
$$F_{gy} = -F_g(\cos\theta)$$
$$= -(562\text{ N})(\cos 30.0°) \qquad \text{Substitute } F_g = 562,\ \theta = 30.0°$$
$$= -487\text{ N}$$

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3 Evaluate the Answer

- **Are the units correct?** Force is measured in newtons.
- **Do the signs make sense?** The components point in directions opposite to the positive axes.
- **Are the magnitudes realistic?** The values are less than F_g.

▶ PRACTICE Problems

Additional Problems, Appendix B

33. An ant climbs at a steady speed up the side of its anthill, which is inclined 30.0° from the vertical. Sketch a free-body diagram for the ant.

34. Scott and Becca are moving a folding table out of the sunlight. A cup of lemonade, with a mass of 0.44 kg, is on the table. Scott lifts his end of the table before Becca does, and as a result, the table makes an angle of 15.0° with the horizontal. Find the components of the cup's weight that are parallel and perpendicular to the plane of the table.

35. Kohana, who has a mass of 50.0 kg, is at the dentist's office having her teeth cleaned, as shown in **Figure 5-14.** If the component of her weight perpendicular to the plane of the seat of the chair is 449 N, at what angle is the chair tilted?

36. Fernando, who has a mass of 43.0 kg, slides down the banister at his grandparents' house. If the banister makes an angle of 35.0° with the horizontal, what is the normal force between Fernando and the banister?

37. A suitcase is on an inclined plane. At what angle, relative to the vertical, will the component of the suitcase's weight parallel to the plane be equal to half the perpendicular component of its weight?

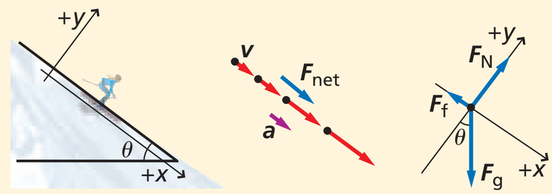
■ **Figure 5-14**

EXAMPLE Problem 6

Skiing Downhill A 62-kg person on skis is going down a hill sloped at 37°. The coefficient of kinetic friction between the skis and the snow is 0.15. How fast is the skier going 5.0 s after starting from rest?

1 Analyze and Sketch the Problem

- Establish a coordinate system.
- Draw a free-body diagram showing the skier's velocity and direction of acceleration.
- Draw a motion diagram showing increasing v, and both a and F_{net} in the $+x$ direction, like the one shown in Figure 5-13.



Known:

$m = 62$ kg
$\theta = 37°$
$\mu_k = 0.15$
$v_i = 0.0$ m/s
$t = 5.0$ s

Unknown:

$a = ?$
$v_f = ?$

2 Solve for the Unknown

y-direction:

$$F_{net,\,y} = ma_y = 0.0 \text{ N}$$

There is no acceleration in the y-direction, so $a_y = 0.0$ m/s^2.

Solve for F_N.

$$F_N - F_{gy} = F_{net,\,y}$$

F_{gy} is negative. It is in the negative direction as defined by the coordinate system.

$$F_N = F_{gy}$$

Substitute $F_{net,\,y} = 0.0$ N and rearrange

$$= mg(\cos\theta)$$

Substitute $F_{gy} = mg\cos\theta$

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page 845

x-direction:

Solve for a.

$$F_{net,\,x} = F_{gx} - F_f$$

F_f is negative because it is in the negative direction as defined by the coordinate system.

$$ma_x = mg(\sin\theta) - \mu_k F_N$$

Substitute $F_{net,\,x} = ma$, $F_{gx} = mg\sin\theta$, $F_f = \mu_k F_N$

$$= mg(\sin\theta) - \mu_k mg(\cos\theta)$$

Substitute $a = a_x$ because all the acceleration is in the x-direction; substitute $F_N = mg\cos\theta$

$$a = g(\sin\theta - \mu_k\cos\theta)$$

$$= (9.80 \text{ m/s}^2)(\sin 37° - (0.15)\cos 37°)$$

Substitute $g = 9.80$ m/s^2, $\theta = 37°$, $\mu_k = 0.15$

$$= 4.7 \text{ m/s}^2$$

Because v_i, a, and t are all known, use the following.

$$v_f = v_i + at$$

$$= 0.0 + (4.7 \text{ m/s}^2)(5.0 \text{ s})$$

Substitute $v_i = 0.0$ m/s, $a = 4.7$ m/s^2, $t = 5.0$ s

$$= 24 \text{ m/s}$$

3 Evaluate the Answer

- **Are the units correct?** Performing dimensional analysis on the units verifies that v_f is in m/s and a is in m/s^2.
- **Do the signs make sense?** Because v_f and a are both in the $+x$ direction, the signs do make sense.
- **Are the magnitudes realistic?** The velocity is fast, over 80 km/h (50 mph), but 37° is a steep incline, and the friction between the skis and the snow is not large.

PRACTICE Problems

Additional Problems, Appendix B

38. Consider the crate on the incline in Example Problem 5. Calculate the magnitude of the acceleration. After 4.00 s, how fast will the crate be moving?

39. If the skier in Example Problem 6 were on a 31° downhill slope, what would be the magnitude of the acceleration?

40. Stacie, who has a mass of 45 kg, starts down a slide that is inclined at an angle of 45° with the horizontal. If the coefficient of kinetic friction between Stacie's shorts and the slide is 0.25, what is her acceleration?

41. After the skier on the 37° hill in Example Problem 6 had been moving for 5.0 s, the friction of the snow suddenly increased and made the net force on the skier zero. What is the new coefficient of friction?

The most important decision in problems involving motion along a slope is what coordinate system to use. Because an object's acceleration is usually parallel to the slope, one axis, usually the *x*-axis, should be in that direction. The *y*-axis is perpendicular to the *x*-axis and perpendicular to the surface of the slope. With this coordinate system, you now have two forces, the normal and frictional forces, in the directions of the coordinate axes; however, the weight is not. This means that when an object is placed on an inclined plane, the magnitude of the normal force between the object and the plane will usually not be equal to the object's weight.

You will need to apply Newton's laws once in the *x*-direction and once in the *y*-direction. Because the weight does not point in either of these directions, you will need to break this vector into its *x*- and *y*-components before you can sum your forces in these two directions. Example Problem 5 and Example Problem 6 both showed this procedure.

MINI LAB

What's Your Angle?

Prop a board up so that it forms an inclined plane at a 45° angle. Hang a 500-g object from the spring scale.

1. **Measure** and record the weight of the object. Set the object on the bottom of the board and slowly pull it up the inclined plane at a constant speed.
2. **Observe and record** the reading on the spring scale.

Analyze and Conclude

3. **Calculate** the component of weight for the 500-g object that is parallel to the inclined plane.
4. **Compare** the spring-scale reading along the inclined plane with the component of weight parallel to the inclined plane.

5.3 Section Review

42. Forces One way to get a car unstuck is to tie one end of a strong rope to the car and the other end to a tree, then push the rope at its midpoint at right angles to the rope. Draw a free-body diagram and explain why even a small force on the rope can exert a large force on the car.

43. Mass A large scoreboard is suspended from the ceiling of a sports arena by 10 strong cables. Six of the cables make an angle of 8.0° with the vertical while the other four make an angle of 10.0°. If the tension in each cable is 1300.0 N, what is the scoreboard's mass?

44. Acceleration A 63-kg water skier is pulled up a 14.0° incline by a rope parallel to the incline with a tension of 512 N. The coefficient of kinetic friction is 0.27. What are the magnitude and direction of the skier's acceleration?

45. Equilibrium You are hanging a painting using two lengths of wire. The wires will break if the force is too great. Should you hang the painting as shown in **Figures 5-15a** or **5-15b?** Explain.

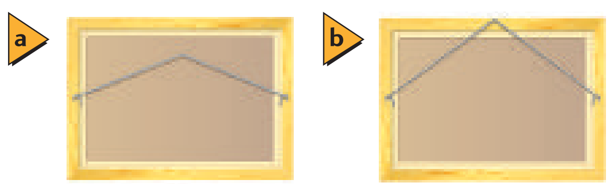


Figure 5-15

46. Critical Thinking Can the coefficient of friction ever have a value such that a skier would be able to slide uphill at a constant velocity? Explain why or why not. Assume there are no other forces acting on the skier.

PHYSICS LAB

Alternate CBL instructions can be found on the Web site.
physicspp.com

The Coefficient of Friction

Static and kinetic friction are forces that are a result of two surfaces in contact with each other. Static friction is the force that must be overcome to cause an object to begin moving, while kinetic friction occurs between two objects in motion relative to each other. The kinetic friction force, $F_{f,\,kinetic}$, is defined by $F_{f,\,kinetic} = \mu_k F_N$, where μ_k is the coefficient of kinetic friction and F_N is the normal force acting on the object. The maximum static frictional force, $F_{f,\,max\,static}$, is defined by $F_{f,\,static} = \mu_s F_N$ where μ_s is the coefficient of static friction and F_N is the normal force on the object. The maximum static frictional force that must be overcome before movement is able to begin is $\mu_s F_N$. If you apply a constant force to pull an object along a horizontal surface at a constant speed, then the frictional force opposing the motion is equal and opposite to the applied force, F_p. Therefore, $F_p = F_f$. The normal force is equal and opposite to the object's weight when the object is on a horizontal surface and the applied force is horizontal.

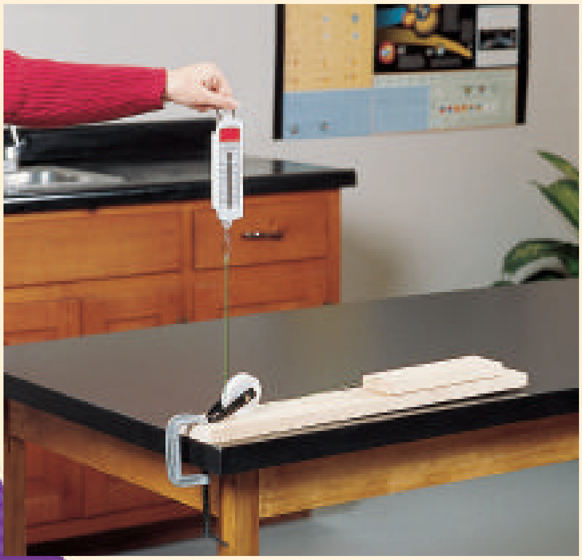
QUESTION

How can the coefficient of static and kinetic friction be determined for an object on a horizontal surface?

Objectives

- **Measure** the normal and frictional forces acting on an object starting in motion and already in motion.
- **Use numbers** to calculate μ_s and μ_k.
- **Compare and contrast** values of μ_s and μ_k.
- **Analyze** the kinetic friction results.
- **Estimate** the angle where sliding will begin for an object on an inclined plane.

Safety Precautions



Materials

pulley
C-clamp
masking tape
wood surface
string (1 m)
spring scale, 0-5 N
wood block

Procedure

1. Check your spring scale to make sure that it reads zero when it is held vertically. If necessary, follow your teacher's instructions to zero it.
2. Attach the pulley to the edge of the table with a C-clamp.
3. Attach the string to the spring scale hook and the wood block.
4. Measure the weight of the block of wood, or other small object, and record the value as the normal force, F_N, in Data Tables 1, 2, and 3.
5. Unhook the string from the spring scale and run it through the pulley. Then reattach it to the spring scale.
6. Move the wood block as far away from the pulley as the string permits, while having it remain on the wood surface.
7. With the spring scale oriented vertically so that a right angle is formed between the wood block, the pulley, and the spring scale, slowly pull up on the spring scale. Observe the force that is necessary to cause the wood block to begin sliding. Record this value for the static frictional force in Data Table 1.

Horizons Companies
CONTENTS

Material Table

Object material	
Surface material	

Data Table 1

F_N (N)	Static Friction Force, F_s (N)			
	Trial 1	Trial 2	Trial 3	Average

Data Table 2

F_N (N)	Kinetic Friction Force, F_f (N)			
	Trial 1	Trial 2	Trial 3	Average

Data Table 3

F_N (N)	F_s (N)	F_f (N)	μ_s	μ_k

Data Table 4 (Angle, θ, when sliding begins on an incline)

θ*	tan θ*

8. Repeat steps 6 and 7 for two additional trials.
9. Repeat steps 6 and 7. However, once the block begins sliding, pull just hard enough to keep it moving at a constant speed across the other horizontal surface. Record this force as the kinetic frictional force in Data Table 2.
10. Repeat step 9 for two additional trials.
11. Place the block on the end of the surface. Slowly raise one end of the surface to make an incline. Gently tap the block to cause it to move and overcome static friction. If the block stops, replace it at the top of the incline and repeat the procedure. Continue increasing the angle, θ, between the horizontal and the inclined surface, and tapping the block until it slides at a constant speed down the incline. Record the angle, θ, in Data Table 4.

Analyze

1. Average the data for the static frictional force, $F_{s, max}$, from the three trials and record the result in the last column of Data Table 1 and in Data Table 3.
2. Average the data for the kinetic frictional force, F_f, from the three trials and record the result in the last column of Data Table 2 and in Data Table 3.
3. Use the data in Data Table 3 to calculate the coefficient of static friction, μ_s, and record the value in Data Table 3.
4. Use the data in Data Table 3 to calculate the coefficient of kinetic friction, μ_k, and record the value in Data Table 3.
5. Calculate tan θ for your value in Data Table 4.

Conclude and Apply

1. **Compare and Contrast** Examine your values for μ_s and μ_k. Explain whether your results are reasonable or not.
2. **Use Models** Draw a free-body diagram showing the forces acting on the block if it is placed on an incline of angle θ. Make certain that you include the force due to friction in your diagram.
3. From your diagram, assuming that the angle, θ, is where sliding begins, what does tan θ represent?
4. Compare your value for tan θ (experimental), μ_s, and μ_k.

Going Further

Repeat the experiment with additional surfaces that have different characteristics.

Real-World Physics

If you were downhill skiing and wished to determine the coefficient of kinetic friction between your skis and the slope, how could you do this? Be specific about how you could find a solution to this problem.

Physics Online

To find out more about friction, visit the Web site: **physicspp.com**

Technology and Society

Roller Coasters

Why are roller coasters fun? A roller-coaster ride would be no fun at all if not for the forces acting on the coaster car and the rider. What forces do riders experience as they ride a roller coaster? The force of gravity acts on the rider and the coaster car in the downward direction. The seat of the car exerts a force on the rider in the opposite direction. When the coaster car makes a turn, the rider experiences a force in the opposite direction. Also, there are forces present due to the friction between the rider and the seat, the side of the car, and the safety bar.

The Force Factor

Designers of roller coasters take into account the magnitude of the forces exerted on the rider. They design the coaster in such a way that the forces thrill the rider without causing injury or too much discomfort.

Designers measure the amount of force exerted on the rider by calculating the force factor. The force factor is equal to the force exerted by the seat on the rider divided by the weight of the rider. Suppose the rider weighs about 68 kg. When the roller coaster is at the bottom of a hill, the rider may experience a force factor of 2. That means that at the bottom of the hill, the rider will feel as though he or she weighs twice as much, or in this case 136 kg. Conversely, at the top of a hill the force factor may be 0.5 and the rider will feel as though he or she weighs half his or her normal weight. Thus, designers create excitement by designing portions that change the rider's apparent weight.

The thrill of a roller-coaster ride is produced by the forces acting on the rider and the rider's reaction to visual cues.

The Thrill Factors Roller-coaster designers manipulate the way in which the body perceives the external world to create that "thrilling" sensation. For example, the roller coaster moves up the first hill very slowly, tricking the rider into thinking that the hill is higher than it is.

The organs of the inner ear sense the position of the head both when it is still and when it is moving. These organs help maintain balance by providing information to the brain. The brain then sends nerve impulses to the skeletal muscles to contract or relax to maintain balance. The constant change in position during a roller-coaster ride causes the organs of the inner ear to send conflicting messages to the brain. As a result, the skeletal muscles contract and relax throughout the ride.

You know that you are moving at high speeds because your eyes see the surroundings move past at high speed. So, designers make use of the surrounding landscape along with twists, turns, tunnels, and loops to give the rider plenty of visual cues. These visual cues, along with the messages from the inner ear, can result in disorientation and in some cases, nausea. To enthusiasts the disorientation is part of the thrill.

In order to attract visitors, amusement parks are constantly working on designing new rides that take the rider to new thrill levels. As roller-coaster technology improves, your most thrilling roller-coaster ride may be over the next hill.

Going Further

1. **Compare and Contrast** Compare and contrast your experience as a rider in the front of a roller coaster versus the back of it. Explain your answer in terms of the forces acting on you.
2. **Critical Thinking** While older roller coasters rely on chain systems to pull the coaster up the first hill, newer ones depend on hydraulic systems to do the same job. Research each of these two systems. What do you think are the advantages and disadvantages of using each system?

Study Guide

5.1 Vectors

Vocabulary

- components (p. 122)
- vector resolution (p. 122)

Key Concepts

- When two vectors are at right angles, you can use the Pythagorean theorem to determine the magnitude of the resultant vector.

$$R^2 = A^2 + B^2$$

- The law of cosines and law of sines can be used to find the magnitude of the resultant of any two vectors.

$$R^2 = A^2 + B^2 - 2AB\cos\theta$$
$$\frac{R}{\sin\theta} = \frac{A}{\sin a} = \frac{B}{\sin b}$$

- The components of a vector are projections of the component vectors.

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{A_x}{A};\ \text{therefore, } A_x = A\cos\theta$$
$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{A_y}{A};\ \text{therefore, } A_y = A\sin\theta$$
$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

- Vectors can be summed by separately adding the *x*- and *y*-components.

5.2 Friction

Vocabulary

- kinetic friction (p. 126)
- static friction (p. 126)
- coefficient of kinetic friction (p. 127)
- coefficient of static friction (p. 127)

Key Concepts

- A frictional force acts when two surfaces touch.
- The frictional force is proportional to the force pushing the surfaces together.
- The kinetic friction force is equal to the coefficient of kinetic friction times the normal force.

$$F_{\text{f, kinetic}} = \mu_k F_N$$

- The static friction force is less than or equal to the coefficient of static friction times the normal force.

$$F_{\text{f, static}} \le \mu_k F_N$$

5.3 Force and Motion in Two Dimensions

Vocabulary

- equilibrant (p. 131)

Key Concepts

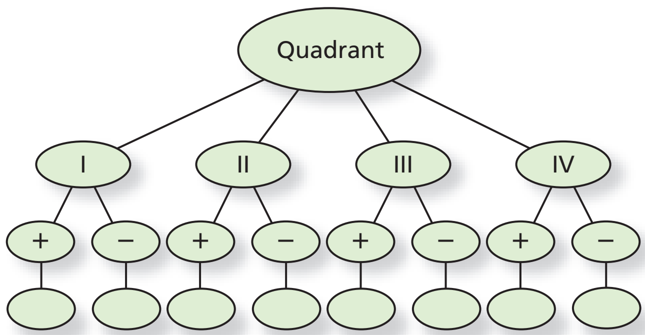
- The force that must be exerted on an object to cause it to be in equilibrium is called the equilibrant.
- The equilibrant is found by finding the net force on an object, then applying a force with the same magnitude but opposite direction.
- An object on an inclined plane has a component of the force of gravity in a direction parallel to the plane; the component can accelerate the object down the plane.

Chapter 5

Assessment

Concept Mapping

47. Complete the concept map below with the terms *sine*, *cosine*, or *tangent* to indicate whether each function is positive or negative in each quadrant. Some circles could remain blank, and others can have more than one term.



Mastering Concepts

48. How would you add two vectors graphically? (5.1)

49. Which of the following actions is permissible when you graphically add one vector to another: moving the vector, rotating the vector, or changing the vector's length? (5.1)

50. In your own words, write a clear definition of the resultant of two or more vectors. Do not explain how to find it; explain what it represents. (5.1)

51. How is the resultant displacement affected when two displacement vectors are added in a different order? (5.1)

52. Explain the method that you would use to subtract two vectors graphically. (5.1)

53. Explain the difference between **A** and *A*. (5.1)

54. The Pythagorean theorem usually is written $c^2 = a^2 + b^2$. If this relationship is used in vector addition, what do a, b, and c represent? (5.1)

55. When using a coordinate system, how is the angle or direction of a vector determined with respect to the axes of the coordinate system? (5.1)

56. What is the meaning of a coefficient of friction that is greater than 1.0? How would you measure it? (5.2)

57. **Cars** Using the model of friction described in this textbook, would the friction between a tire and the road be increased by a wide rather than a narrow tire? Explain. (5.2)

58. Describe a coordinate system that would be suitable for dealing with a problem in which a ball is thrown up into the air. (5.3)

59. If a coordinate system is set up such that the positive x-axis points in a direction 30° above the horizontal, what should be the angle between the x-axis and the y-axis? What should be the direction of the positive y-axis? (5.3)

60. Explain how you would set up a coordinate system for motion on a hill. (5.3)

61. If your textbook is in equilibrium, what can you say about the forces acting on it? (5.3)

62. Can an object that is in equilibrium be moving? Explain. (5.3)

63. What is the sum of three vectors that, when placed tip to tail, form a triangle? If these vectors represent forces on an object, what does this imply about the object? (5.3)

64. You are asked to analyze the motion of a book placed on a sloping table. (5.3)
   a. Describe the best coordinate system for analyzing the motion.
   b. How are the components of the weight of the book related to the angle of the table?

65. For a book on a sloping table, describe what happens to the component of the weight force parallel to the table and the force of friction on the book as you increase the angle that the table makes with the horizontal. (5.3)
   a. Which components of force(s) increase when the angle increases?
   b. Which components of force(s) decrease?

Applying Concepts

66. A vector that is 1 cm long represents a displacement of 5 km. How many kilometers are represented by a 3-cm vector drawn to the same scale?

67. **Mowing the Lawn** If you are pushing a lawn mower across the grass, as shown in **Figure 5-16**, can you increase the horizontal component of the force that you exert on the mower without increasing the magnitude of the force? Explain.

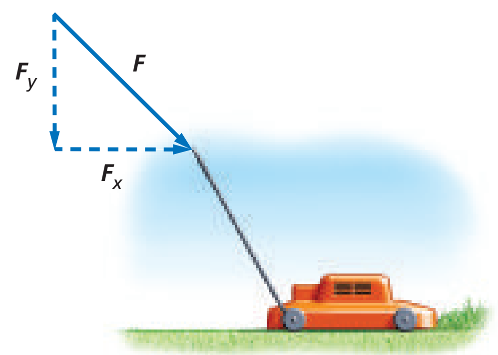


Figure 5-16

68. A vector drawn 15 mm long represents a velocity of 30 m/s. How long should you draw a vector to represent a velocity of 20 m/s?

69. What is the largest possible displacement resulting from two displacements with magnitudes 3 m and 4 m? What is the smallest possible resultant? Draw sketches to demonstrate your answers.

70. How does the resultant displacement change as the angle between two vectors increases from 0° to 180°?

71. A and B are two sides of a right triangle, where $\tan \theta = A/B$.
a. Which side of the triangle is longer if $\tan \theta$ is greater than 1.0?
b. Which side is longer if $\tan \theta$ is less than 1.0?
c. What does it mean if $\tan \theta$ is equal to 1.0?

72. Traveling by Car A car has a velocity of 50 km/h in a direction 60° north of east. A coordinate system with the positive x-axis pointing east and a positive y-axis pointing north is chosen. Which component of the velocity vector is larger, x or y?

73. Under what conditions can the Pythagorean theorem, rather than the law of cosines, be used to find the magnitude of a resultant vector?

74. A problem involves a car moving up a hill, so a coordinate system is chosen with the positive x-axis parallel to the surface of the hill. The problem also involves a stone that is dropped onto the car. Sketch the problem and show the components of the velocity vector of the stone.

75. Pulling a Cart According to legend, a horse learned Newton's laws. When the horse was told to pull a cart, it refused, saying that if it pulled the cart forward, according to Newton's third law, there would be an equal force backwards; thus, there would be balanced forces, and, according to Newton's second law, the cart would not accelerate. How would you reason with this horse?

76. Tennis When stretching a tennis net between two posts, it is relatively easy to pull one end of the net hard enough to remove most of the slack, but you need a winch to take the last bit of slack out of the net to make the top almost completely horizontal. Why is this true?

77. The weight of a book on an inclined plane can be resolved into two vector components, one along the plane, and the other perpendicular to it.
a. At what angle are the components equal?
b. At what angle is the parallel component equal to zero?
c. At what angle is the parallel component equal to the weight?

78. TV Towers The transmitting tower of a TV station is held upright by guy wires that extend from the top of the tower to the ground. The force along the guy wires can be resolved into two perpendicular components. Which one is larger?

Mastering Problems

5.1 Vectors

79. Cars A car moves 65 km due east, then 45 km due west. What is its total displacement?

80. Find the horizontal and vertical components of the following vectors, as shown in **Figure 5-17.**
a. E
b. F
c. A

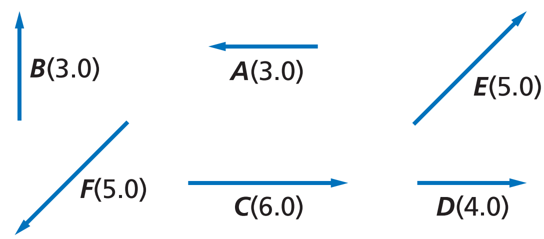


Figure 5-17

81. Graphically find the sum of the following pairs of vectors, whose lengths and directions are shown in Figure 5-17.
a. D and A
b. C and D
c. C and A
d. E and F

82. Graphically add the following sets of vectors, as shown in Figure 5-17.
a. A, C, and D
b. A, B, and E
c. B, D, and F

83. You walk 30 m south and 30 m east. Find the magnitude and direction of the resultant displacement both graphically and algebraically.

84. Hiking A hiker's trip consists of three segments. Path A is 8.0 km long heading 60.0° north of east. Path B is 7.0 km long in a direction due east. Path C is 4.0 km long heading 315° counterclockwise from east.
a. Graphically add the hiker's displacements in the order A, B, C.
b. Graphically add the hiker's displacements in the order C, B, A.
c. What can you conclude about the resulting displacements?

85. What is the net force acting on the ring in **Figure 5-18?**

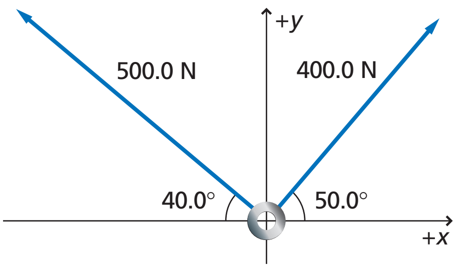


Figure 5-18

86. What is the net force acting on the ring in **Figure 5-19?**

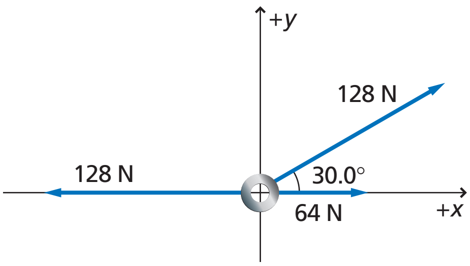


Figure 5-19

87. A Ship at Sea A ship at sea is due into a port 500.0 km due south in two days. However, a severe storm comes in and blows it 100.0 km due east from its original position. How far is the ship from its destination? In what direction must it travel to reach its destination?

88. Space Exploration A descent vehicle landing on Mars has a vertical velocity toward the surface of Mars of 5.5 m/s. At the same time, it has a horizontal velocity of 3.5 m/s.

a. At what speed does the vehicle move along its descent path?

b. At what angle with the vertical is this path?

89. Navigation Alfredo leaves camp and, using a compass, walks 4 km E, then 6 km S, 3 km E, 5 km N, 10 km W, 8 km N, and, finally, 3 km S. At the end of three days, he is lost. By drawing a diagram, compute how far Alfredo is from camp and which direction he should take to get back to camp.

5.2 Friction

90. If you use a horizontal force of 30.0 N to slide a 12.0-kg wooden crate across a floor at a constant velocity, what is the coefficient of kinetic friction between the crate and the floor?

91. A 225-kg crate is pushed horizontally with a force of 710 N. If the coefficient of friction is 0.20, calculate the acceleration of the crate.

92. A force of 40.0 N accelerates a 5.0-kg block at 6.0 m/s^2 along a horizontal surface.

a. How large is the frictional force?

b. What is the coefficient of friction?

93. Moving Appliances Your family just had a new refrigerator delivered. The delivery man has left and you realize that the refrigerator is not quite in the right position, so you plan to move it several centimeters. If the refrigerator has a mass of 180 kg, the coefficient of kinetic friction between the bottom of the refrigerator and the floor is 0.13, and the static coefficient of friction between these same surfaces is 0.21, how hard do you have to push horizontally to get the refrigerator to start moving?

94. Stopping at a Red Light You are driving a 2500.0-kg car at a constant speed of 14.0 m/s along a wet, but straight, level road. As you approach an intersection, the traffic light turns red. You slam on the brakes. The car's wheels lock, the tires begin skidding, and the car slides to a halt in a distance of 25.0 m. What is the coefficient of kinetic friction between your tires and the wet road?

5.3 Force and Motion in Two Dimensions

95. An object in equilibrium has three forces exerted on it. A 33.0-N force acts at 90.0° from the *x*-axis and a 44.0-N force acts at 60.0° from the *x*-axis. What are the magnitude and direction of the third force?

96. Five forces act on an object: (1) 60.0 N at 90.0°, (2) 40.0 N at 0.0°, (3) 80.0 N at 270.0°, (4) 40.0 N at 180.0°, and (5) 50.0 N at 60.0°. What are the magnitude and direction of a sixth force that would produce equilibrium?

97. Advertising Joe wishes to hang a sign weighing 7.50×10^2 N so that cable *A*, attached to the store, makes a 30.0° angle, as shown in **Figure 5-20.** Cable *B* is horizontal and attached to an adjoining building. What is the tension in cable *B*?

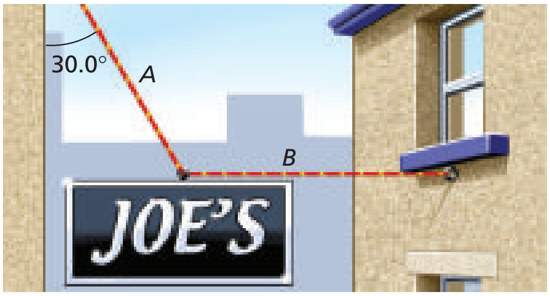


Figure 5-20

 For more problems, go to Additional Problems, Appendix B.

98. A street lamp weighs 150 N. It is supported by two wires that form an angle of 120.0° with each other. The tensions in the wires are equal.

a. What is the tension in each wire supporting the street lamp?

b. If the angle between the wires supporting the street lamp is reduced to 90.0°, what is the tension in each wire?

99. A 215-N box is placed on an inclined plane that makes a 35.0° angle with the horizontal. Find the component of the weight force parallel to the plane's surface.

100. Emergency Room You are shadowing a nurse in the emergency room of a local hospital. An orderly wheels in a patient who has been in a very serious accident and has had severe bleeding. The nurse quickly explains to you that in a case like this, the patient's bed will be tilted with the head downward to make sure the brain gets enough blood. She tells you that, for most patients, the largest angle that the bed can be tilted without the patient beginning to slide off is 32.0° from the horizontal.

a. On what factor or factors does this angle of tilting depend?

b. Find the coefficient of static friction between a typical patient and the bed's sheets.

101. Two blocks are connected by a string over a frictionless, massless pulley such that one is resting on an inclined plane and the other is hanging over the top edge of the plane, as shown in **Figure 5-21.** The hanging block has a mass of 16.0 kg, and the one on the plane has a mass of 8.0 kg. The coefficient of kinetic friction between the block and the inclined plane is 0.23. The blocks are released from rest.

a. What is the acceleration of the blocks?

b. What is the tension in the string connecting the blocks?

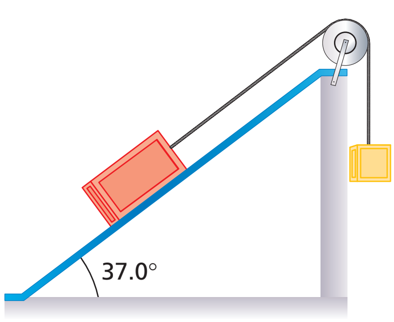


Figure 5-21

102. In **Figure 5-22,** a block of mass *M* is pushed with a force, *F*, such that the smaller block of mass *m* does not slide down the front of it. There is no friction between the larger block and the surface below it, but the coefficient of static friction between the two blocks is μ_s. Find an expression for *F* in terms of *M*, *m*, μ_s, and *g*.

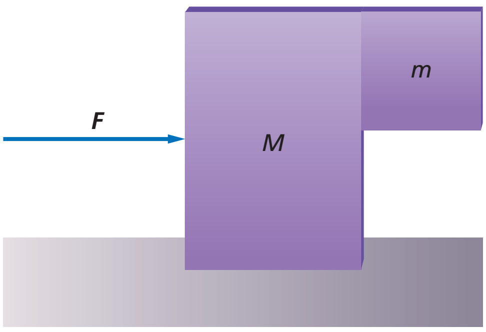


Figure 5-22

Mixed Review

103. The scale in **Figure 5-23** is being pulled on by three ropes. What net force does the scale read?

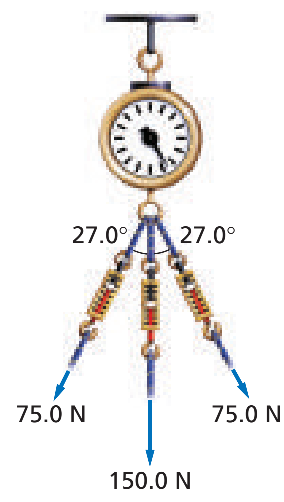


Figure 5-23

104. Sledding A sled with a mass of 50.0 kg is pulled along flat, snow-covered ground. The static friction coefficient is 0.30, and the kinetic friction coefficient is 0.10.

a. What does the sled weigh?

b. What force will be needed to start the sled moving?

c. What force is needed to keep the sled moving at a constant velocity?

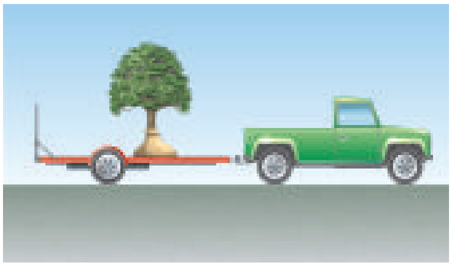
d. Once moving, what total force must be applied to the sled to accelerate it at 3.0 m/s^2?

105. Mythology Sisyphus was a character in Greek mythology who was doomed in Hades to push a boulder to the top of a steep mountain. When he reached the top, the boulder would slide back down the mountain and he would have to start all over again. Assume that Sisyphus slides the boulder up the mountain without being able to roll it, even though in most versions of the myth, he rolled it.

a. If the coefficient of kinetic friction between the boulder and the mountainside is 0.40, the mass of the boulder is 20.0 kg, and the slope of the mountain is a constant 30.0°, what is the force that Sisyphus must exert on the boulder to move it up the mountain at a constant velocity?

b. If Sisyphus pushes the boulder at a velocity of 0.25 m/s and it takes him 8.0 h to reach the top of the mountain, what is the mythical mountain's vertical height?

106. Landscaping A tree is being transported on a flatbed trailer by a landscaper, as shown in **Figure 5-24.** If the base of the tree slides on the trailer, the tree will fall over and be damaged. If the coefficient of static friction between the tree and the trailer is 0.50, what is the minimum stopping distance of the truck, traveling at 55 km/h, if it is to accelerate uniformly and not have the tree slide forward and fall on the trailer?



■ **Figure 5-24**

Thinking Critically

107. Use Models Using the Example Problems in this chapter as models, write an example problem to solve the following problem. Include the following sections: Analyze and Sketch the Problem, Solve for the Unknown (with a complete strategy), and Evaluate the Answer. A driver of a 975-kg car traveling 25 m/s puts on the brakes. What is the shortest distance it will take for the car to stop? Assume that the road is concrete, the force of friction of the road on the tires is constant, and the tires do not slip.

108. Analyze and Conclude Margaret Mary, Doug, and Kako are at a local amusement park and see an attraction called the Giant Slide, which is simply a very long and high inclined plane. Visitors at the amusement park climb a long flight of steps to the top of the 27° inclined plane and are given canvas sacks. They sit on the sacks and slide down the 70-m-long plane. At the time when the three friends walk past the slide, a 135-kg man and a 20-kg boy are each at the top preparing to slide down. "I wonder how much less time it will take the man to slide down than it will take the boy," says Margaret Mary. "I think the boy will take less time," says Doug. "You're both wrong," says Kako. "They will reach the bottom at the same time."

a. Perform the appropriate analysis to determine who is correct.

b. If the man and the boy do not take the same amount of time to reach the bottom of the slide, calculate how many seconds of difference there will be between the two times.

Writing in Physics

109. Investigate some of the techniques used in industry to reduce the friction between various parts of machines. Describe two or three of these techniques and explain the physics of how they work.

110. Olympics In recent years, many Olympic athletes, such as sprinters, swimmers, skiers, and speed skaters, have used modified equipment to reduce the effects of friction and air or water drag. Research a piece of equipment used by one of these types of athletes and the way it has changed over the years. Explain how physics has impacted these changes.

Cumulative Review

111. Add or subtract as indicated and state the answer with the correct number of significant digits. (Chapter 1)

a. 85.26 g + 4.7 g

b. 1.07 km + 0.608 km

c. 186.4 kg − 57.83 kg

d. 60.08 s − 12.2 s

112. You ride your bike for 1.5 h at an average velocity of 10 km/h, then for 30 min at 15 km/h. What is your average velocity? (Chapter 3)

113. A 45-N force is exerted in the upward direction on a 2.0-kg briefcase. What is the acceleration of the briefcase? (Chapter 4)

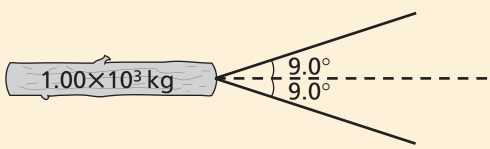
 For more problems, go to Additional Problems, Appendix B.

Standardized Test Practice

Multiple Choice

1. Two tractors pull against a 1.00×10^3-kg log. If the angle of the tractors' chains in relation to each other is 18.0°, and each tractor pulls with a force of 8×10^2 N, what forces will they be able to exert?

- (A) 250 N
- (B) 1.52×10^3 N
- (C) 1.58×10^3 N
- (D) 1.60×10^3 N



2. An airplane pilot tries to fly directly east with a velocity of 800.0 km/h. If a wind comes from the southwest at 80.0 km/h, what is the relative velocity of the airplane to the surface of Earth?

- (A) 804 km/h, 5.7° N of E
- (B) 858 km/h, 3.8° N of E
- (C) 859 km/h, 4.0° N of E
- (D) 880 km/h 45° N of E

3. For a winter fair, some students decide to build 30.0-kg wooden pull-carts on sled skids. If two 90.0-kg passengers get in, how much force will the puller have to exert to move a pull-cart? The coefficient of maximum static friction between the cart and the snow is 0.15.

- (A) 1.8×10^2 N
- (B) 3.1×10^2 N
- (C) 2.1×10^3 N
- (D) 1.4×10^4 N

4. It takes a minimum force of 280 N to move a 50.0-kg crate. What is the coefficient of maximum static friction between the crate and the floor?

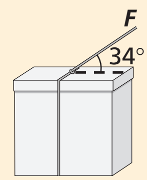
- (A) 0.18
- (B) 0.57
- (C) 1.8
- (D) 5.6

5. What is the y-component of a 95.3-N force that is exerted at 57.1° to the horizontal?

- (A) 51.8 N
- (B) 80.0 N
- (C) 114 N
- (D) 175 N

6. A string exerts a force of 18 N on a box at an angle of 34° from the horizontal. What is the horizontal component of the force on the box?

- (A) 10 N
- (B) 15 N
- (C) 21.7 N
- (D) 32 N



7. Sukey is riding her bicycle on a path when she comes around a corner and sees that a fallen tree is blocking the way 42 m ahead. If the coefficient of friction between her bicycle's tires and the gravel path is 0.36, and she is traveling at 50.0 km/h, how much stopping distance will she require? Sukey and her bicycle, together, have a mass of 95 kg.

- (A) 3.00 m
- (B) 4.00 m
- (C) 8.12 m
- (D) 27.3 m

Extended Answer

8. A man starts from a position 310 m north of his car and walks for 2.7 min in a westward direction at a constant velocity of 10 km/h. How far is he from his car when he stops?

9. Jeeves is tired of his 41.2-kg son sliding down the banister, so he decides to apply an extremely sticky paste that increases the coefficient of static friction to 0.72 to the top of the banister. What will be the magnitude of the static friction force on the boy if the banister is at an angle of 52.4° from the horizontal?

✓ Test-Taking TIP

Calculators Are Only Machines

If your test allows you to use a calculator, use it wisely. Figure out which numbers are relevant, and determine the best way to solve the problem before you start punching keys.